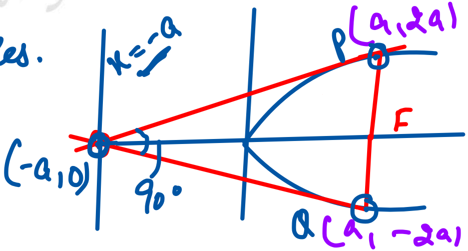
Plane Geometry

Parabola

Important Questions (PYQ)

Prove that the tangents at the end of a latus rectum of a parabola intersect on the directrix at right angles.



Sol. Tangent at Point P $(a, 2a)$

$$yy_1 = 2a(x + x_1)$$

$$2ay = 2a(x + a)$$

$$x - y + a = 0 \quad \text{---} \; (1)$$

eq. of Tangent at point Q $(a, -2a)$

is $$yy_1 = 2a(x + x_1)$$

$$y(-2a) = 2a(x + a)$$

$$x + y + a = 0 \quad \text{--- (II)}$$

Add (I) & (II)

$$x - \cancel{y} + a + x + \cancel{y} + a = 0$$

$$2x + 2a = 0$$

$$x = -a.$$

Put $x = -a$ in (I)

$$y = 0$$

Point of intersection of tangents $(-a, 0)$

which lies on diretrix — 3

Slope of 1 $m_1 = 1$

Slope of 2 $m_2 = -1.$

$m_1 m_2 = -1.$ —

$\therefore$ Tangents are $\perp$. — 4

Hence from 3 & 4

Tangents at ends of latus rectum of

a Parabola intersect on the directrix at right angles.

Hence Proved.

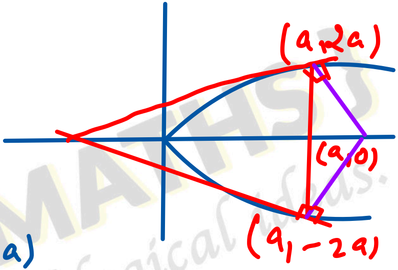
(2) Show that normals at the extrimities of a latus rectum of a Parabola intersect at right angles on the axis of Parabola.

Sol. Let Parabola

$$y^2 = 4ax.$$

end Points of Latus Rectum $P(a, 2a)$

$Q(a, -2a)$



Normal at Point $P(a, 2a)$ is

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

$$y - 2a = \frac{-2a}{2a}(x - a)$$

$$y - 2a = -x + a$$

$$y + x - 3a = 0 \quad \text{— (i)}$$

Normal at Point Q $(a, -2a)$

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

$$y - (-2a) = \frac{-(-2a)}{2a}(x - a)$$

$$y + 2a = x - a$$

$$y - x + 3a = 0 \quad \text{— (ii)}$$

Add (I) & (II)

$$y + \cancel{x} - 3\cancel{a} + y - \cancel{x} + 3\cancel{a} = 0$$

$$y = 0$$

Put $y = 0$ in (II)

$$x = 3a.$$

Point of intersection of (I) & (II)

$(3a, 0)$ which lies on x axis

or axis of Parabola. — (3)

Slope of (1) $m_1 = -1$

slope of (2) $m_2 = 1$

$$m_1 m_2 = -1$$

$\therefore$ (1) & (2) are $\perp$

$\Rightarrow$ Normals intersect at right angle — (4)

from (3) and (4)

The normals at the extrimities of the latus rectum of the parabola intersect at right angles on axis of Parabola.