Plane Geometry

Pair Of Straight Lines

Full Chapter Revision and PYQ'S

☆ $(ax+by+c)(a'x+b'y+c')=0$

represents a pair of lines

$ax+by+c=0$

$a'x+b'y+c'=0$

☆ The second degree homogenous equation

$ax^2+2hxy+by^2=0$

- a pair of distinct lines through origin

$$h^2 - ab > 0$$

- a line or coincide lines through origin

$$h^2 - ab = 0$$

- an empty set if $h^2 - ab < 0$

☆ $ax^2 + 2hxy + by^2 = 0$ represent two lines through origin.

$y = m_1 x \qquad y = m_2 x.$

$$bm^2 + 2hm + a = 0$$

$$m_1 + m_2 = \frac{-2h}{b}$$

$x = 1$

$y = m$.

$$m_1 m_2 = \frac{a}{b}.$$

☆ θ between pair of lines

$$ax^2 + 2hxy + by^2 = 0$$

$$\boxed{\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|}$$

- The lines $ax^2 + 2hxy + by^2 = 0$ are perpendicular when $a + b = 0$

Coeff. of x^2 + Coeff of $y^2 = 0$

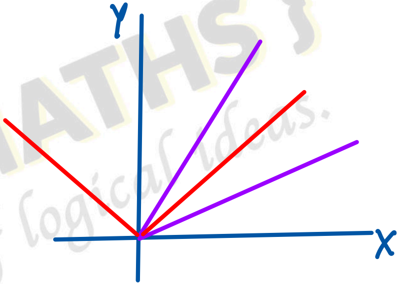
- The lines $ax^2 + 2hxy + by^2 = 0$ are Coincide when

$h^2 - ab = 0$

★ The Joint equation of straight lines Bisecting the angles between lines

$ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h}.$$



General second degree equation.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represent the straight lines

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

✪ $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent pair of lines then joint equation of lines through origin and parallel to these lines

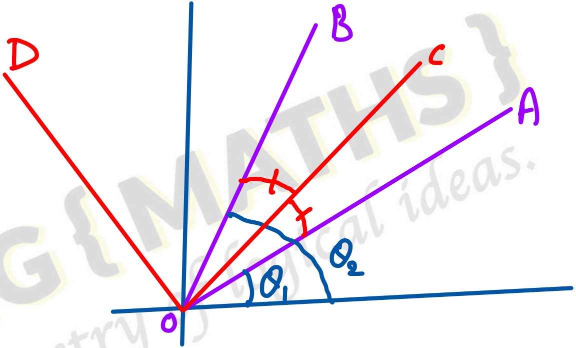
$$ax^2 + 2hxy + by^2 = 0.$$

① Find the equation to the straight lines bisecting the angles between the straight lines given by $ax^2 + 2hxy + by^2 = 0$

Sol: Let OA and OB are two lines $y = m_1 x$ and $y = m_2 x$

$$m_1 = \tan\theta_1$$

$$m_2 = \tan\theta_2$$



$$m_1 + m_2 = \frac{-2h}{b}.$$

$$m_1 m_2 = \frac{a}{b}.$$

$$\left.\begin{aligned} \tan\theta_1 + \tan\theta_2 &= \frac{-2h}{b} \\ \tan\theta_1 \tan\theta_2 &= a/b \end{aligned}\right] \quad -(1)$$

Let OC is internal and OD is External Bisector of $\angle AOB$.

Now

$$\angle AOC = \angle COB$$

$$\angle XOC - \theta_1 = \theta_2 - \angle XOC$$

$$2\angle XOC = \theta_1 + \theta_2$$

$$\angle XOC = \frac{\theta_1 + \theta_2}{2}$$

$\angle XOD = \angle XOC + \angle COD$

$= \dfrac{\theta_1+\theta_2}{2} + \pi/2.$

[Angle between interior and exterior Bisector is always $90°$]

Let θ be the angle made by bisectors with x-axis

either $\theta = \dfrac{\theta_1+\theta_2}{2}$ or $\theta = \dfrac{\theta_1+\theta_2}{2} + \pi/2$

either $2\theta = \theta_1 + \theta_2$ or $2\theta = \theta_1 + \theta_2 + \pi$

$$\tan 2\theta = \tan(\theta_1 + \theta_2)$$

$$= \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

$$= \frac{-\frac{2h}{b}}{1 - \frac{a}{b}}$$ [from (1)]

$$= \frac{-2h}{b-a} = \frac{2h}{a-b}$$

$$\tan 2\theta = \frac{2h}{a-b}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2h}{a-b} \quad — (2)$$

equation of bisector is $y = \tan\theta\, x$.

$\tan\theta = y/x$

Put value of $\tan\theta$ in (2).

$$\frac{2\,y/x}{1-y^2/x^2}=\frac{2h}{a-b}$$

$$\frac{y}{x\,\frac{(x^2-y^2)}{x^2}}=\frac{h}{a-b}$$

$$\frac{xy}{x^2-y^2}=\frac{h}{a-b}.$$

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}.$$

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