

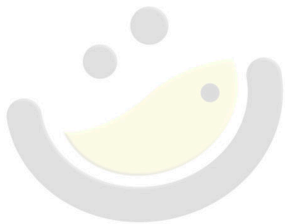
Calculus

Limit and Continuity : Important Questions

Show that the function.

$$f(x) = \begin{cases} \frac{e^{\sqrt{x}} - 1}{e^{\sqrt{x}} + 1} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is Discontinuous at $x=0$



Sol.

$$\lim_{x \rightarrow 0^-} \frac{e^{1/x} - 1}{e^{1/x} + 1}$$

$$x = 0 - h, \quad h > 0$$

$$x \rightarrow 0^- \quad \underline{\underline{h \rightarrow 0}}$$

$$\lim_{h \rightarrow 0} \frac{e^{-1/h} - 1}{e^{-1/h} + 1} = \frac{0 - 1}{0 + 1} = -1.$$

$$\lim_{x \rightarrow 0^-} f(x) = -1 \quad \text{--- (i)}$$

$$\lim_{x \rightarrow 0^+} \frac{e^{1/x} - 1}{e^{1/x} + 1}$$

$$x = 0+h, \quad h > 0 \\ x \rightarrow 0^+ \quad h \rightarrow 0$$

$$\lim_{h \rightarrow 0} \frac{e^{1/h} - 1}{e^{1/h} + 1}$$

Divide numerator and denominator with $e^{1/h}$.

$$\lim_{h \rightarrow 0} \frac{\frac{e^{1/h}}{e^{1/h}} - \frac{1}{e^{1/h}}}{e^{1/h} \left(\frac{e^{1/h}}{e^{1/h}} + \frac{1}{e^{1/h}} \right)}$$

$$\lim_{h \rightarrow 0} \frac{1 - e^{-|h|}}{1 + e^{|h|}} = \frac{1 - 0}{1 + 0} = 1.$$

$$\lim_{x \rightarrow 0^+} f(x) = 1. \quad \text{--- (ii)}$$

from (i) + (ii)

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$\therefore f(x)$ is discontinuous at $x=0$.

hence proved.

②

Prove that $\lim_{x \rightarrow a} \frac{1}{x-a}$ does not exist.

③

Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist.

