

Calculus

Limit and Continuity : Important Questions

For What values of a and b will the following functions be continuous at x ?

$$f(x) = \begin{cases} 13 \\ ax^2 + bx + 1 \\ 17 - ax \end{cases}$$



$$x \leq 2$$

$$2 < x < 3$$

$$x \geq 3$$



Sol.

f is continuous for all x .

$\therefore f$ is continuous for $x = 2, 3$. — ①

$\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$ [from ①].

$$13 = \lim_{x \rightarrow 2^+} ax^2 + bx + 1$$

$$\lim_{h \rightarrow 0} a(2+h)^2 + b(2+h) + 1 = 13.$$

$$\begin{aligned} x &= 2+h \\ x &\rightarrow 2^+ \\ h &\rightarrow 0 \end{aligned}$$

$$4a + 2b + 1 = 13.$$

$$4a + 2b = 12$$

$$2a + b = 6$$

→ (11)

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) \quad [\text{from } \textcircled{1}]$$

$$\lim_{x \rightarrow 3^-} ax^2 + bx + 1 = \lim_{x \rightarrow 3^+} 17 - ax$$

$$x = 3 - h$$
$$x \rightarrow 3^-$$
$$h \rightarrow 0$$

$$\lim_{h \rightarrow 0}$$

$$a(3-h)^2 + b(3-h) + 1$$

$$= \lim_{h \rightarrow 0} 17 - a(3+h)$$
$$x = 3 + h.$$
$$x \rightarrow 3^+$$
$$h \rightarrow 0$$

$$9a + 3b + 1 = 17 - 3a$$

$$9a + 3b + 3a = 17 - 1$$

$$12a + 3b = 16$$

— (iii)

$$\left. \begin{array}{l} 2a + b = 6 \\ 12a + 3b = 16 \end{array} \right\} \begin{array}{l} \times 6 \\ \times 1 \end{array}$$

$$\begin{array}{r} 12a + 6b = 36 \\ \underline{12a + 3b = 16} \\ \hline \end{array}$$

$$3b = 20$$

$$b = \frac{20}{3}$$

Put $b = \frac{20}{3}$ in (11)

$$2a + \frac{20}{3} = 6$$

$$2a = 6 - \frac{20}{3}$$

$$= \frac{18 - 20}{3} = \frac{-2}{3}$$

$$2a = \frac{-2}{3}$$

$$a = \frac{-1}{3}$$

$$b = \frac{20}{3}$$

Ans.

$$a = -\frac{1}{3}$$



OMG { MATHS }
The poetry of logical ideas.