Calculus
Limit and Continuity : Important Questions Prove that the function.

$$
f(x)=\left\{\begin{array}{cc}
\frac{x}{|x|+2 x^{2}} & x \neq 0 \\
k & x=0
\end{array}\right.
$$remains discontinuous at $x=0$ regardless of the choice of $k$.

So $\quad \lim _{x \rightarrow 0^{-}} f(x)=\lim _{x \rightarrow 0^{-}} \frac{x}{|x|+2 x^{2}}$

$$
\lim _{x \rightarrow 0^{-}} \frac{x}{-x+2 x^{2}}
$$



$$
\begin{equation*}
\lim _{x \rightarrow 0^{-}} \frac{x}{x(-1+2 x)}=-1 \tag{1}
\end{equation*}
$$

$$
\begin{aligned}
& \begin{aligned}
\lim _{x \rightarrow 0^{+}} f(x) & =\lim _{x \rightarrow 0^{+}} \frac{x}{|x|+2 x^{2}} \left\lvert\, \begin{array}{l}
x \rightarrow 0^{+} \\
\\
\\
=\lim _{x \rightarrow 0^{+}} \frac{x}{x+2 x^{2}} \\
x>0 \text { slignten. } \\
|x|=x .
\end{array}\right.,
\end{aligned} \\
& =\lim _{x \rightarrow 0^{+}} \frac{x x}{x(1+2 x)} \\
& =1 . \quad \text { - (II) }
\end{aligned}
$$

from (1) \& (11)

$$
\lim _{x \rightarrow 0^{-}} f(x) \neq \operatorname{dim}_{x \rightarrow 0^{+}} f(x)
$$

$\Rightarrow \quad f(x)$ is discontinuous at $x=0$.
regardless of choice of $t$ Hence Proved.

