# Calculus

## Limit and Continuity : Important Questions

Prove that the function

$$f(x) = \begin{cases} \dfrac{x}{|x| + 2x^2} & x \neq 0 \\ K & x = 0 \end{cases}$$

remains discontinous at $x = 0$ regardless of the choice of $K$.

Sol:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x| + 2x^2}$$

$$\lim_{x\to 0^-} \frac{x}{-x + 2x^2}$$

$$\lim_{x\to 0^-} \frac{\cancel{x}}{\cancel{x}(-1 + 2x)} = -1. \quad \text{—— (1)}$$

0

$x \to 0^-$

$x < 0$ slightly

$|x| = -x.$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{|x| + 2x^2}$$

$$= \lim_{x \to 0^+} \frac{x}{x + 2x^2}$$

$$= \lim_{x \to 0^+} \frac{\cancel{x}}{\cancel{x}(1 + 2x)}$$

$$= 1. \quad \text{— (ii)}$$

$x \to 0^+$

$0$

$x > 0$ slightly

$|x| = x.$

from (i) & (ii)

$$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

$\Rightarrow$ $f(x)$ is discontinous at $x = 0$.

regardless of choice of $k$

Hence Proved.