

# Calculus

## Limit and Continuity : Important Questions

$$\text{Let } f(x) = \begin{cases} 1 & x \leq 3 \\ ax + b & 3 < x < 5 \\ 7 & 5 \leq x \end{cases}$$

Determine the constants  $a$  and  $b$  so that  $f$  may be continuous for all  $x$ .

Sol.  $f$  is continuous for all  $x$  (given)

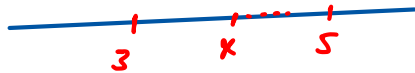
$\therefore f$  is continuous for  $3, 5$ .

$$\Rightarrow \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3) \quad \text{--- (i)}$$

also  $\Rightarrow \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = f(5) \quad \text{--- (ii)}$

$$\lim_{x \rightarrow 3^+} f(x) = f(3) \quad (\text{from (i)})$$

$$\lim_{x \rightarrow 3^+} ax + b = 1$$



$$\lim_{h \rightarrow 0} a(3+h) + b = 1$$

$$x = 3+h. \quad h > 0$$

$$3a + b = 1$$

— (ii)

$$x \rightarrow 3^+$$

$$h \rightarrow 0.$$

$$\lim_{x \rightarrow 5^-} f(x) = f(5) \quad (\text{from (i)})$$

$$\lim_{x \rightarrow 5^-} ax + b = 7$$

$$\lim_{h \rightarrow 0} a(5-h) + b = 7$$

$$x = 5-h$$

$$x \rightarrow 5^-$$

$$h \rightarrow 0.$$

$$5a + b = 7 \quad - \textcircled{iv}$$

subtract  $\textcircled{iii}$  from  $\textcircled{iv}$

$$5a + \cancel{b} - 3a - \cancel{b} = 7 - 1$$

$$2a = 6$$

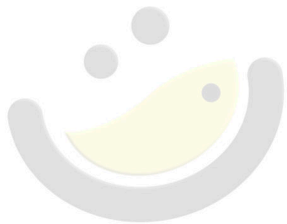
$$a = 3.$$

Put  $a = 3$  in  $\textcircled{iv}$

$$15 + b = 7$$

$$b = 7 - 15 = -8 \text{ Ans.}$$

for  $a = 3$  and  $b = -8$   $f$  is continuous.



**OMG { MATHS }**  
The poetry of logical ideas.