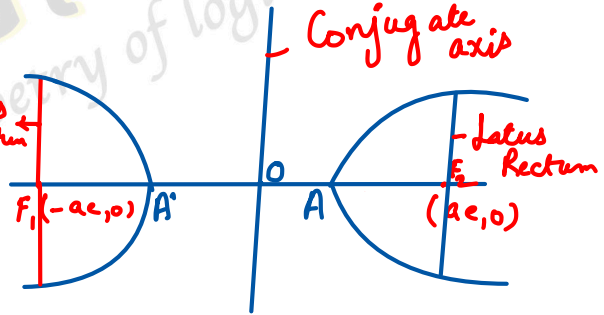
Plane Geometry
Hyperbola
Chapter Revision

Standard form of Hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

F_1 & F_2 → two focus

A & A' → two vertex



Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Transverse axis	
equation	$y = 0$
length	$2a$.
Conjugate axis	
equation	$x = 0$
length	$2b$.
Focus (F_1)	$(ae, 0)$
(F_2)	$(-ae, 0)$

Latus Rectum equations	$x = \pm ae$
Directrix	$x = a/e$ & $x = -a/e$
Length of Latus Rectum	$\frac{2b^2}{a}$
Centre	$(0,0)$
Asymptotes	$y = \pm \frac{b}{a} x$

Rectangular hyperbola

Length of transverse and conjugate axis will be equal

eg. $x^2 - y^2 = a^2$

eccentricity = $\sqrt{2}$

equation of asymptotes — $x + y = 0$, $x - y = 0$

Conjugate Hyperbola

Two hyperbolas s.t. transverse and Conjugate axis of one are respectively conjugate and transverse axis of other. are called Conjugate hyperbola

Exp $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ are Conjugate.

Equation of tangent

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

equation of Normal

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2+b^2$$

Condition of tangency

Line $lx + my + n = 0$ is tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if

$$a^2 l^2 - b^2 m^2 = n^2$$

$y = mx + c$ is tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if

$$c^2 = a^2 m^2 - b^2$$

∴ Equation of tangent to hyperbola in slope form.

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

Condition of Normality

$lx + my + n = 0$ is normal to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if

$$\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$$

Chord of Contact

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Equation of Polar

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Equation of Pair of tangents

$$\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}-1\right)\left(\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1\right)=\left(\frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1\right)^2$$

Pole of line $lx+my+n=0$

$$\left(-\frac{a^2l}{n},\ -\frac{b^2m}{n}\right)$$

Equation of chord in terms of mid point

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}$$