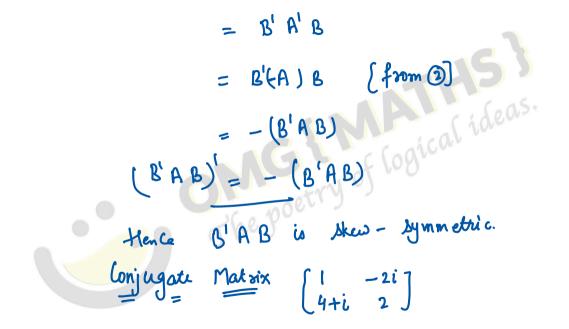


A'= - A Skew Symmetric Matrix & Show that the Matrin B'AB is symmetric or skew symmetric acording as A is symmetric or skew symmetric. 1000 A is Symmetric A' = A - 0(B'AB)' = B'A'(B')'

= B' A' B E:(B')'= B [from 0] = B'AB f logical ideas. (B'AB)' = B'ABCaseTI (B'AB)' = B'A'(B')'

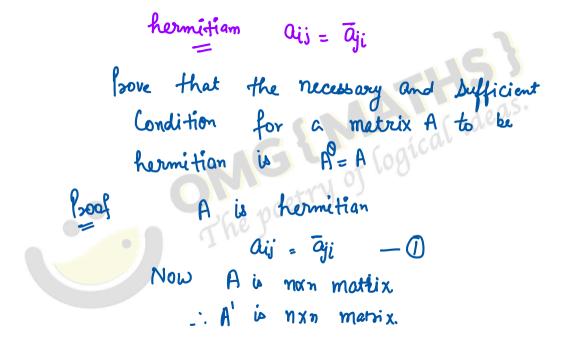


 $\overline{A} = \begin{bmatrix} 1 & 2i \\ 4-i & 2 \end{bmatrix}$ Tranjugate Matrix $A = \begin{bmatrix} 1 & -2i \\ 4+i & 2 \end{bmatrix}$ fogical ideas. $A' = \begin{bmatrix} 1 & 4+i \\ -2i & 2 \end{bmatrix}$ $\mathbf{A}^{0} = (\mathbf{A}^{\prime}) = \begin{bmatrix} 1 & 4-i \\ 2i & 2 \end{bmatrix}$

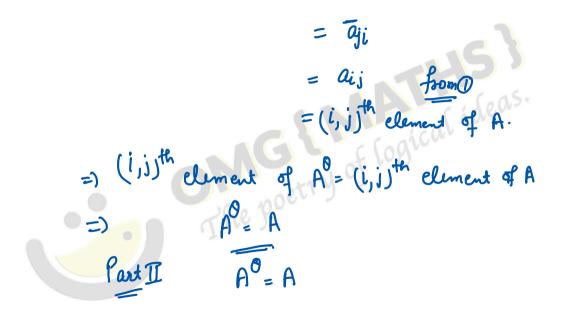
Is $(\overline{A})' = (\overline{A}')$ Justify your Answer Yes. Justification $A = \begin{pmatrix} 1+i & 3i \\ -4 & 0 & 1-i \end{pmatrix}$ $L \cdot H \cdot s = \begin{bmatrix} \overline{A} \end{bmatrix} = \begin{bmatrix} \overline{A} = \begin{bmatrix} 1 - i & -3i \\ 4 & 1 + i \end{bmatrix}$ $\left(\overline{A}\right) = \begin{bmatrix} 1-i & 4\\ -3i & 1+i \end{bmatrix} - 0$

 $A = \begin{pmatrix} 1+i & 3i \\ 4 & 1-i \end{pmatrix}$ f logical ideas. $A' = \begin{pmatrix} 1+i & 4 \\ 3i & 1-i \end{pmatrix}$ $(\overline{A'}) = \begin{pmatrix} 1-i & 4 \\ -3i & 1+i \end{pmatrix} - 0$ from 0 + 10 $(\overline{A})' = (\overline{A'})$ Hence Proved.

Hermitian M atri x Aquare A° = A. Skew hermitian matrix $A = \begin{bmatrix} 1 & 4 + 3i \\ 4 - 3i & 5 \end{bmatrix}$ ₽° - −A $A' = \begin{bmatrix} r & 4-3i \\ 4+3i & 5 \end{bmatrix}$ $a_{12} = 4 + 3i$ Q21= 4-31 $(\overline{A'}) = \begin{pmatrix} 1 & 4+3i \\ 4-3i & 5 \end{pmatrix} = A \overline{a_{21}} = 4+3i \\ a_{12} = \overline{a_{21}} \\ a_{13} = \overline{a_{33}}$



: (A) is nxn matrix : A is nxn matrix : A" + A is of same type (i, j) the element of A = Complex Conjugate of Now (i, j)the element of A' = Complex Conjugate of (i,i)th element of A = Complex Conjugate of Aji



La A is mxn matrix. A' is nom matrix = $(\overline{A'})$ is nxm matrix. But $\overline{A'} = \overline{A}$. $\cdot A'' f A$ is of same type. $A^0 = (\overline{A'})$ is norm matrix. ... n= m. A is non speare matrix.

$$A^{0} = A$$

=) $(i, j)^{th}$ element of $A^{0} = (i, j)^{th}$ element of A
=) $(\text{omplex (onjugate of (i, j)^{th} element of A})$
 $= (i, j)^{th}$ element of A
=) $(\text{omplex (onjugate of (j, i)^{th} element of A})$
 $= (i, j)^{th}$ element of A
 $= (i, j)^{th}$ element of A

=) A is Hermitian. A and B are hermition. Show that AB+ BA is hermitian and AB-BA is skew-hermitian. A and B are hermitian (Given) Sol The. , A0 = A 80= B. -0 $(AB + BA)^{\circ} = (AB)^{\circ} + (BA)^{\circ}$ = $B^{\circ}A^{\circ} + A^{\circ}B^{\circ}$

= BA + AB [from 0] = AB + BA AB+BA is hermitian $(AB-BA)^{\circ} = (AB)^{\circ} - (BA)^{\circ}$ $= R^{\circ} A^{\circ} - A^{\circ} B^{\circ}$ = BA - AB [from 0] = -(AB-BA)

 $(AB-BA)^2 = - (AB-BA)$ Hence AB-BA is skew-hermitian. Show that every share matrix Canbe expressed in one and only one Way as a sum of a hermitian and a skew - hermitian matrix,

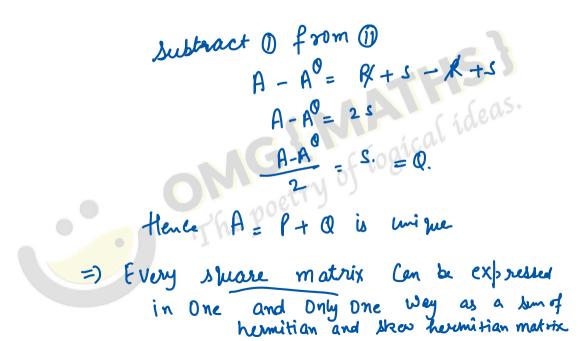
Let A is a sprare matrix 501. $A = \frac{A}{2} + \frac{A}{2}$ $+ \frac{A^0}{2} 2 \frac{A^0}{2}$ $=\frac{A}{2}$ + $= 1 (A + A^{0}) + 1 (A - A^{0})$ P + QA

 $l = \frac{1}{2} \left(A + A^{0} \right)$ $P^{0} = \frac{1}{2} (A + A^{0})^{0} = \frac{1}{2} (A^{0} + (A^{0})^{0})$ $= \frac{1}{2} (A^{0} + A) = \frac{1}{2} (A + A^{0})$ mition. -

 $Q = \frac{1}{2} \left[A - A^{0} \right]$ $Q^{0} = \frac{1}{2} [A - A^{0}]^{0}$ $= \frac{1}{2} \left[\left(A^{0} - (A^{0})^{0} \right) \right] 0 gical ideas.$ $= \frac{1}{2} \left(A^{0} - A \right) = -\frac{1}{2} \left(A - A^{0} \right) = -Q.$ -Q. Skew - hermitian matrix.

A= 1+9 It skew hermitian a sprare matrix can be expressed Alence in hermitian and skew hermitian matrix Let + 5 where R is hermitianded Skew - hermitian 2 i.

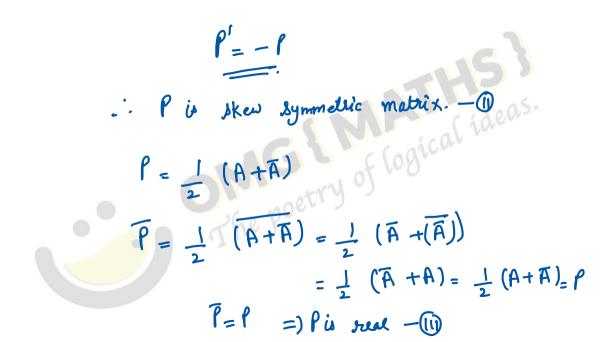
 $R = R \qquad S^0 = -S.$ S' = S' = S' = $A^{0} = R^{0} + S^{0}$ $A^{0} = R - S - 0$ A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 A = R + S - 0 $\frac{A + A^0}{2} = R = P$



Every share Matrix Can be expressed in One and only one way as Ptil where P and Q are hermitian Do it yoursey. & show that every skew hermitian metrix. A Canbe uniquely expressed as P-id where P - real skew symmetric Q - Real Symmetric.

is skew hermitian matrix 300 $\therefore A^{\circ} = -A$ A ideas. $A = \frac{A}{2} + \frac{A}{2} + \frac{A}{2} - \frac{A}{2} -$ $= \underbrace{1(A + A)}_{2} - \underbrace{1xi}_{2i} (A + A)$ -iQwhere $P = \frac{1}{2} (A + \overline{A}) Q = \frac{1}{2i} (\overline{A} - A)$

 $l = \frac{1}{2} (A + \overline{A})$ $P' = \frac{1}{2} (A + \overline{A})' = \frac{1}{2} [A' + (\overline{A})']$ $= \sqrt{2} \left[A' + A' \right]$ $=\frac{1}{2}(-A^{0})' + A^{0}$ $=\frac{1}{2}(-((\bar{A})')'-\bar{A})(from 0)$ $= \frac{1}{2} \left[- \overline{A} - A \right] = -\frac{1}{2} (A + \overline{A})$



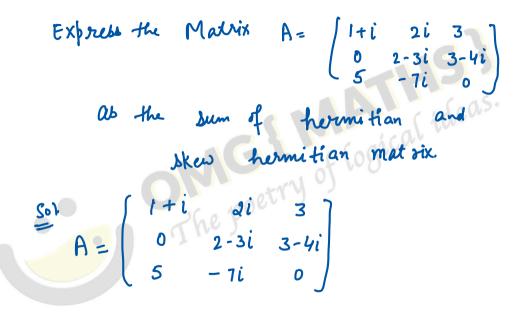
from (1) and (1) l is real skew symmetric Matrix. Q= 1 (A-A) 2i (A-A) of logical ideas. $Q' = \frac{1}{2i} (\overline{A} - A)' = \frac{1}{2i} [(\overline{A})' - A']$ $= \frac{1}{2!} \left[\left[A^{0} + \left(A^{0} \right)^{\prime} \right] \right] \left(from 0 \right)$

 $=\frac{1}{2i}\left[-A + ((\bar{A})')'\right]_{a}$ $= \frac{1}{2i} \left[-A + \overline{A} \right] = \frac{1}{2i} \left(\overline{A} - A \right) = 0.$ Q' = Q.Q is symmetric Matrix. - 1 Hence $\overline{Q} = \left(\frac{1}{2i} (\overline{A} - A) \right) = \frac{-1}{2i} (\overline{A} - A)$ $= \frac{1}{2i} (\overline{A}) - \overline{A})$

 $= \frac{-1}{2i} (A - \overline{A})$ $(\overline{A} - A) = Q.$ Q=Q. =) Q is real - () (and O. from Q is real symmetric Matrix A Can be expressed in form of P-iQ where Pis real skew symmetric and

Q is real symmetric Matrix we have to prove A = P-iQ is Now anifue. Ris real sken symmetric where A= R-is. _0 det s is real symmetric A = R - is= R+15 $\vec{R} = R$ $\overline{A} = R + is. - (n)$

Add (VI) and (VI) $A + \overline{A} = 2R$ subtract () from ()) And 000000 ()) A-A . . = Q. A= R-is is same as A= P-iO. Hence representation is cuiture



 $A' = \begin{cases} 1+i & 0 & 5 \\ 2i & 2-3i & -7i \\ 3 & 3-4i & 0 \end{cases}$ $A^{\circ} = (A^{\prime}) = \begin{cases} 1 - i & 0 & 5 \\ -ai & 2 + 3i & 7i \\ 3 & 3 + 4i & 0 \end{cases}$

 $= \begin{cases} 2 & 2i & 8 \\ -2i & 4 & 3+3i \\ 8 & 3-3i & 0 \end{cases}$ $\frac{A + A^{\circ}}{2} = 1$ (3|z+3|zi) which is hermitian i 0 - 6p 2001

Hence
$$A = \begin{pmatrix} A + A^0 \\ 2 \end{pmatrix} + \begin{pmatrix} A - A^0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & i & 4 \\ -i & 2 & 3|_2 + 3|_2 i \\ 4 & 3|_2 - 3|_2 i & 0 \end{pmatrix} + \begin{pmatrix} i & i & -1 \\ i & -3i & 3|_2 - 11|_2 i \\ 1 & -3h - 11|_2 i & 0 \end{pmatrix}$$

$$\therefore Matrix A has been expressed as the sum of hermitian Matrix.$$