Calculus

General Theorems: Important Questions

Apply Taylor's Thm to prove that

$$e^{\cos x} = 1 - (x - \pi/2) + \frac{(x-\pi/2)^2}{2} - \frac{(x-\pi/2)^4}{8} + \text{------}$$

Taylor's Thm

$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \frac{(b-a)^3}{3!}f'''(a) + \text{----}$$

$$f(x) = f(\pi/2) + (x-\pi/2) f'(\pi/2) + \frac{(x-\pi/2)^2}{2!} f''(\pi/2) \quad [\pi/2, x]$$

$$+ \frac{(x-\pi/2)^3}{3!} f'''(\pi/2) + \cdots \quad \text{—(1)}$$

$$f(x) = e^{\cos x}$$

$$f(x) = e^{\cos x} \qquad f(\pi/2) = e^{\cos \pi/2} = e^0 = 1 \quad \text{—(2)}$$

$$f'(x) = e^{\cos x}(-\sin x) \qquad f'(\pi/2) = e^0(-1) = -1 \quad \text{—(3)}$$

$$f''(x) = e^{\cos x}(-\cos x) + (-\sin x)e^{\cos x}(-\sin x)$$

$$= e^{\cos x}\left[-\cos x + \sin^2 x\right]$$

$$f''(\pi/2) = e^{0}\left[0 + 1\right]$$

$$= 1. \quad \text{—④}$$

$$f'''(x) = e^{\cos x}\left[+\sin x + 2\sin x\cos x\right] + (-\cos x + \sin^2 x)\,e^{\cos x}(-\sin x)$$

$$= e^{\cos x}\left[\sin x + 2\cos x\sin x + \sin x\cos x - \sin^3 x\right]$$

$$= e^{\cos x}\left[\sin x + 3\sin x\cos x - \sin^3 x\right]$$

$$f'''(\pi/2) = e^0\left[1 + (-1)\right]$$
$$= e^0[1-1] = 0. \quad —⑤$$

Put 2, 3, 4, 5 in ①

$$f(x) = 1 - \left(x - \pi/2\right) + \frac{\left(x - \pi/2\right)^2}{2} + 0 + \text{------}$$

$$\text{----}$$

$$e^{\cos x} = 1 - \left(x - \pi/2\right) + \frac{\left(x - \pi/2\right)^2}{2} + \text{-------}$$

Hence Proved.

Use Taylor's Thm to express the Polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$

By Taylor's Thm $[2, x]$.

$$f(x) = f(2) + (x-2) f'(2) + \frac{(x-2)^2}{2!} f''(2) + \frac{(x-2)^3}{3!} f'''(2) + \cdots \quad \text{--- (1)}$$

$$f(x) = 2x^3 + 7x^2 + x - 6$$

$$f'(x) = 6x^2 + 14x + 1$$

$$f''(x) = 12x + 14$$

$$f'''(x) = 12$$

$$f(2) = 2 \times 8 + 7 \times 4 + 2 - 6$$
$$= 40. \quad - ②$$

$$f'(2) = 6 \times 4 + 14 \times 2 + 1$$
$$= 24 + 28 + 1$$
$$= 53. \quad - ③$$

$$f''(2) = 12 \times 2 + 14$$
$$= 24 + 14$$
$$= 38 \quad - ④$$

$$f'''(2) = 12 \quad - ⑤$$

Put values of 2, 3, 4, 5 in (1)

$$f(x) = 40 + (x-2)\,53 + \frac{(x-2)^2}{2} \times 38 + \frac{(x-2)^3}{6} \times 12$$

$$= 40 + 53(x-2) + 19(x-2)^2 + 2(x-2)^3$$

Ans.

Expand.

$2 + x^2 - 3x^5 + 7x^6$ in powers of $(x-1)$