

# Calculus

## General Theorems: Important Questions

Apply Taylor's Thm to prove that

$$e^{\cos x} = 1 - \frac{(x - \pi/2)}{2} + \frac{(x - \pi/2)^2}{2} - \frac{(x - \pi/2)^4}{8} +$$

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Taylor's Thm

$$= f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \frac{(b-a)^3}{3!}f'''(a)$$

+ -----



$$f''(x) = e^{\cos x} (-\cos x) + (-\sin x) e^{\cos x} (-\sin x)$$

$$= e^{\cos x} [-\cos x + \sin^2 x]$$

$$f''(\pi/2) = e^0 [0 + 1]$$

$$= 1.$$

④

$$f'''(x) = e^{\cos x} [ +\sin x + 2 \sin x \cos x ]$$

$$+ (-\cos x + \sin^2 x) e^{\cos x} (-\sin x)$$

$$= e^{\cos x} [ \sin x + 2 \cos x \sin x + \sin x \cos x - \sin^3 x ]$$

$$= e^{\cos x} [ \sin x + 3 \sin x \cos x - \sin^3 x ]$$

$$f'''(\pi/2) = e^0 [1 + (-1)] \\ = e^0 [1 - 1] = 0.$$

Put 2, 3, 4, 5 in ①

$$f(x) = 1 - (x - \pi/2) + \frac{(x - \pi/2)^2}{2} + 0 + \dots$$

$$e^{\cos x} = 1 - (x - \pi/2) + \frac{(x - \pi/2)^2}{2} + \dots$$

Hence Proved.

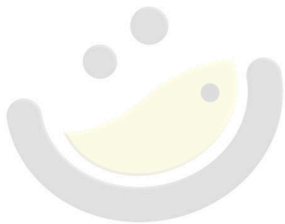
Use Taylor's Thm to express the polynomial.

$2x^3 + 7x^2 + x - 6$  in powers of  $(x-2)$

By Taylor's Thm

$$f(x) = f(2) + (x-2)f'(2) + \frac{(x-2)^2}{2!}f''(2) + \frac{(x-2)^3}{3!}f'''(2) + \dots$$

— (1)



$$f(x) = 2x^3 + 7x^2 + x - 6$$

$$f'(x) = 6x^2 + 14x + 1$$

$$f''(x) = 12x + 14$$

$$f'''(x) = 12$$

$$f(2) = 2 \times 8 + 7 \times 4 + 2 - 6$$

$$= 40. \quad \text{--- (2)}$$

$$f'(2) = 6 \times 4 + 14 \times 2 + 1$$

$$= 24 + 28 + 1$$

$$= 53. \quad \text{--- (3)}$$

$$f''(2) = 12 \times 2 + 14$$

$$= 24 + 14$$

$$= 38 \quad \text{--- (4)}$$

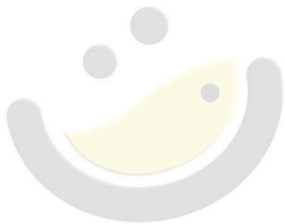
$$f'''(2) = 12 \quad \text{--- (5)}$$

Put values of 2, 3, 4, 5 in ①

$$f(x) = 40 + (x-2)53 + \frac{(x-2)^2}{2} \times 38 + \frac{(x-2)^3}{6} \times 12$$

$$= 40 + 53(x-2) + 19(x-2)^2 + 2(x-2)^3$$

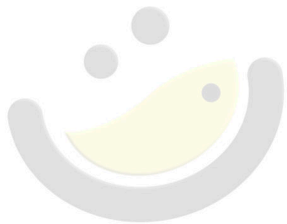
Ans.



OMG! MATHS!  
The poetry of logical ideas.

Expand.

$2 + x^2 - 3x^5 + 7x^6$  in powers of  $(x-1)$



OMG { MATHS }

The poetry of logical ideas.