

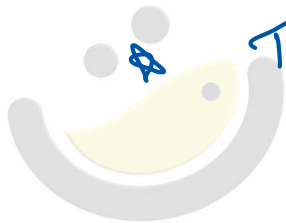
General Theorems: Important Questions

⊛ Rolle's thm.

⊛ Lagrange's thm.

⊛ Cauchy's Mean Value thm.

⊛ Taylor's thm.



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use Cauchy's Mean Value thm. to

evaluate $\lim_{x \rightarrow 1} \frac{\cos \pi x / 2}{\log 1/x}$

Sol.

$$f(x) = \cos \frac{\pi x}{2}$$

$$g(x) = \log x$$

$f(x)$ and $g(x)$ are continuous in $[x, 1]$

$f(x)$ and $g(x)$ are derivable in $(x, 1)$

also $g'(x) = \frac{1}{x} \neq 0$ in $(x, 1)$

\therefore all conditions of Cauchy's mean value thm are satisfied.

$$\Rightarrow \frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

$$b = 1$$
$$a = x$$

$$\Rightarrow \frac{f(1) - f(x)}{g(1) - g(x)} = \frac{f'(c)}{g'(c)}$$

where

$$\underline{\underline{x < c < 1}}$$

$$\Rightarrow \frac{\cos \frac{\pi}{2} - \cos \frac{\pi x}{2}}{\log 1 - \log x} = \frac{-\frac{\pi}{2} \sin \frac{\pi c}{2}}{1/c}$$

$$\Rightarrow \frac{-\cos \frac{\pi x}{2}}{\log 1/x} = \frac{-\frac{\pi}{2} \sin \frac{\pi c}{2}}{1/c}$$

Taking limit both side

$$x < c < 1$$

$$x \rightarrow 1$$

$$c \rightarrow 1$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\cos \frac{\pi x}{2}}{\log \sqrt{x}} = \lim_{c \rightarrow 1} \frac{\pi/2 \sin \frac{\pi c}{2}}{1/c}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\cos \pi x / 2}{\log \sqrt{x}} = \pi/2 \sin \pi/2 \quad [\sin \pi/2 = 1]$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\cos \pi x / 2}{\log \sqrt{x}} = \frac{\pi}{2} \quad \underline{\underline{\text{Ans.}}}$$

②

Using Cauchy's Mean Value theorem

Prove that for $x > 1$

$$\lim_{n \rightarrow \infty} n(x^{1/n} - 1) = \log x$$

Proof

$$f(x) = x^{1/n}$$

$$g(x) = \log x$$

$$f(x)$$

and $g(x)$ are continuous in $[1, x]$

$$f(x)$$

and $g(x)$ are derivable in $(1, x)$

$$f'(x) = \frac{1}{x} \neq 0 \text{ in } (1, x)$$

\therefore all conditions of Cauchy's mean value theorem satisfied

$$\frac{f(x) - f(1)}{g(x) - g(1)} = \frac{f'(c)}{g'(c)} \quad \text{where } 1 < c < x$$

$$\frac{x^{1/n} - 1}{\log x - \log 1} = \frac{\frac{1}{n} c^{\frac{1}{n} - 1}}{c^{-1}}$$

$$\frac{x^{1/n} - 1}{\log x} = \frac{1}{n} c^{\frac{1}{n}}$$

$$n (x^{1/n} - 1) = c^{1/n} \cdot \log x$$

Take $\lim_{n \rightarrow \infty}$ both sides.

$$\lim_{n \rightarrow \infty} n (x^{1/n} - 1) = \log x \lim_{n \rightarrow \infty} c^{1/n}$$

$$\lim_{n \rightarrow \infty} n (x^{1/n} - 1) = \log x.$$

Hence Proved.
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