Calculus
Derivatives of Hyperbolic And Inverse Hyperbolic Functions
Important Questions (pyq)

Differentiate

$$\frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a}$$

Sol.

$$y = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x \cdot \frac{1}{\cancel{2}\sqrt{x^2-a^2}}(\cancel{2}x) + \sqrt{x^2-a^2}(1) \right]$$

$$-\frac{a^2}{2}\left[\frac{1}{\sqrt{x^2\left[a^2-1\right]}} \right] \cdot \frac{1}{a}$$

$$= \frac{1}{2}\left[\frac{x^2}{\sqrt{x^2-a^2}} + \sqrt{x^2-a^2} \right] - \frac{a^2}{2}\left[\frac{1 \times \cancel{a} \cdot 1}{\sqrt{x^2-a^2} \cdot \cancel{a}} \right]$$

$$= \frac{1}{2} \left[\frac{x^2 + x^2 - a^2}{\sqrt{x^2 - a^2}} \right] - \frac{a^2}{2} \cdot \frac{1}{\sqrt{x^2 - a^2}}$$

$$= \frac{1}{2\sqrt{x^2 - a^2}} \left[2x^2 - a^2 - a^2 \right]$$

$$= \frac{1}{2\sqrt{x^2 - a^2}} \left(2(x^2 - a^2) \right) = \sqrt{x^2 - a^2}$$

Ans.

② Differentiate the following w.r.t x.

$$\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

③ Differentiate $\tan^{-1}(\operatorname{sech} x^2)$ w.r.t x^2

④ Prove that $\dfrac{d}{dx} \tanh(\log x) = \dfrac{4x}{(x^2 + 1)^2}$