Plane Geometry

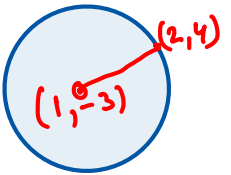
Circle

Important questions (PYQ)

Find the equation of the circle passing through the point (2,4) and which has its centre at intersection of $x - y = 4$ & $2x + 3y = -7$

Sol

$$\begin{matrix} x - y = 4 \\ 2x + 3y = -7 \end{matrix} \Bigg] \begin{matrix} \times 2 \\ \times 1 \end{matrix}$$



$$2x - 2y = 8$$
$$\underline{-2x + 3y = -7}$$
$$-5y = 15$$
$$y = -3.$$

Put $y = -3$ in $x - y = 4$

$$x + 3 = 4$$
$$x = 1$$

Point of intersection of lines is $(1, -3)$

$\therefore$ Centre of Circle $(1, -3)$

Now Radius of Circle is

$$\sqrt{(2-1)^2 + (4-(-3))^2}$$

$$= \sqrt{1 + 49} = \sqrt{50}$$

Eq. of Circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

where r is Radius, (h, k) is Centre of Circle

$$(x-1)^2 + (y+3)^2 = (\sqrt{50})^2$$

$$x^2 + 1 - 2x + y^2 + 9 + 6y = 50$$

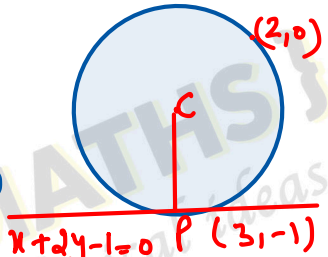
$$x^2 + y^2 - 2x + 6y - 40 = 0.$$

Which is required eq.

(2) Find the eq. of Circle which passes through the point $(2, 0)$ and touches the straight line $x + 2y - 1 = 0$ at the point $(3, -1)$

Sol. Let eq. of Circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad -(1)$$



It passes through $(2,0)$

$$4 + 4g + c = 0$$

$$4g + c = -4. \quad -(ii)$$

eq. (1) also passes through $(3,-1)$

$$9 + 1 + 6g - 2f + c = 0$$

$$6g - 2f + c = -10 \quad -(iii)$$

Subtrace (iii) from (ii)

$$4g + c - 6g + 2f - c = -4 + 10$$

$$-2g + 2f = 6$$

$$2g - 2f + 6 = 0$$

$$g - f + 3 = 0 \quad - (iv)$$

Centre of Circle is $(-g, -f)$

Slope of CP $(m_1) = \dfrac{-1+f}{3+g}$

Now slope of $x+2y-1=0$ is

$$m_2 = -1/2.$$

Also line cf and given line $x+2y-1=0$ are $\perp$ to each other.

$$\therefore m_1 m_2 = -1$$

$$\left(\frac{-1+f}{3+g}\right)\left(\frac{-1}{2}\right) = -1$$

$$\frac{-1+f}{3+g} = 2$$

$$-1+f = 6+2g$$

$$2g - f + 7 = 0$$

$$g - f + 3 = 0 \quad \text{(eq. ②)}$$

$$- \quad + \quad -$$

$$g + 4 = 0$$

$$g = -4.$$

Put $g = -4$ in (iv)

$$-4 - f + 3 = 0$$

$$-f - 1 = 0$$

$$f = -1.$$

Put value of $g = -4$ in (ii)

$$4g + c = -4$$

$$4(-4) + c = -4$$

$$-16 + c = -4$$

$$c = -4 + 16$$

$$c = 12$$

Put value of g, f, c in (1)

$$x^2 + y^2 - 8x - 2y + 12 = 0.$$

Which is the required eq.

③ Find the equation of the circle which passes through the points $(4,1)$ and $(6,5)$ and has its centre on the line

$$4x + y = 16$$

Sol^{n} Let eq of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad -①$$

① passes through $(4,1)$ (Given)

$$16 + 1 + 8g + 2f + c = 0$$

$$8g + 2f + c + 17 = 0 \quad -(II)$$

Also (I) passes through $(6,5)$

$$36 + 25 + 12g + 10f + c = 0$$

$$12g + 10f + c + 61 = 0 \quad -(III)$$

Subtract (II) from (III)

$$12g + 10f + c + 61 - 8g - 2f - c - 17 = 0$$

$$4g + 8f + 44 = 0$$

$$g + 2f + 11 = 0 \quad -(IV)$$

Now $(-g, -f)$ is Centre of Circle.

$(-g, -f)$ lies on the lines

$$4x + y = 16 \quad \text{[Given]}$$

$$-4g - f = 16$$

$$4g + f + 16 = 0 \quad -(v)$$

$$\left.\begin{matrix} g + 2f + 11 = 0 \\ 4g + f + 16 = 0 \end{matrix}\right] \begin{matrix} \times 4 \\ \times 1 \end{matrix}$$

$$4g + 8f + 44 = 0$$

$$-4g + f + 16 = 0$$

$$7f + 28 = 0$$

$$f = \frac{-28}{7} = -4.$$

Put value of f in (iv)

$$g + 2f + 11 = 0$$

$$g - 8 + 11 = 0$$

$$g = -3.$$

Put value of g and f in (ii)

$$c = 15$$

Now Put values of g, f, c in (i)

$$x^2 + y^2 - 6x - 8y + 15 = 0.$$

which is the required equation.