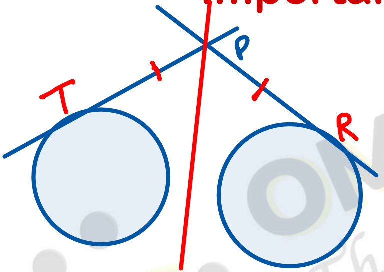
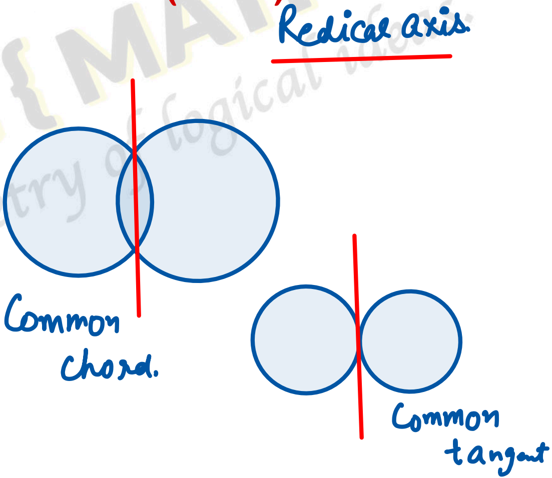
Plane Geometry

Circle

Important questions (PYQ)





Find the radical axis and length of common chord of the circles

$$x^2+y^2+ax+by+c=0 \quad -\text{(I)}$$
$$x^2+y^2+bx+ay+c=0 \quad -\text{(II)}$$

subtract (II) from (I)

$$\cancel{x^2}+\cancel{y^2}+ax+by+\cancel{c}-\cancel{x^2}-\cancel{y^2}-bx-ay-\cancel{c}=0$$

$$ax+by-bx-ay=0$$

$$(a-b)x+(b-a)y=0$$

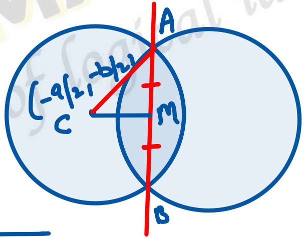
$$(a-b)(x-y)=0$$

$$x-y=0$$

which is required redical axis

Centre of circle ①

$$\left(-\frac{a}{2}, -\frac{b}{2}\right)$$



Radius of ① $\sqrt{g^2+f^2-c}$

$$r = \sqrt{\frac{a^2}{4}+\frac{b^2}{4}-c} = \frac{\sqrt{a^2+b^2-4c}}{2}$$

CM is $\perp$ to line $x - y = 0$

where $C\left(\frac{-a}{2}, \frac{-b}{2}\right)$

$$\therefore \quad CM = \frac{\left|-\frac{a}{2} + \frac{b}{2}\right|}{\sqrt{1+1}}$$

$$\underline{CM} = \frac{|a-b|}{2\sqrt{2}}$$

Also $AB = 2AM$ $\quad$ ($\because$ CM bisects the chord AB)

ΔCAM is right angled Δ.

$$\therefore (CA)^2 = (CM)^2 + (AM)^2 \quad \text{[By Pythagoras]}$$

$$\therefore (AM)^2 = (CA)^2 - (CM)^2$$

$$= \frac{a^2+b^2-4c}{4} - \frac{(a-b)^2}{8}$$

$$= \frac{2a^2+2b^2-8c-a^2-b^2+2ab}{8}$$

$$(AM)^2 = \frac{a^2+b^2-8c+2ab}{8}$$

$$(AM)^2 = \frac{(a+b)^2 - 8c}{8}$$

$$AM = \sqrt{\frac{(a+b)^2 - 8c}{8}}$$

$$= \frac{1}{2}\sqrt{\frac{(a+b)^2 - 8c}{2}}$$

Now length of chord $AB = 2AM$.

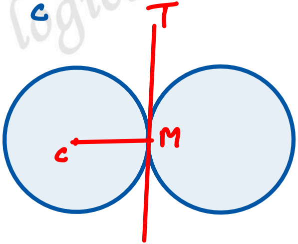
$$\therefore \quad AB = \sqrt{\frac{(a+b)^2 - 8c}{2}}$$

② Prove that two circles

$$x^2+y^2+2ax+c=0 \quad \text{—} (I)$$

$$x^2+y^2+2by+c=0 \quad \text{—} (II)$$

touch if $\dfrac{1}{a^2}+\dfrac{1}{b^2}=\dfrac{1}{c}$



Sol. Subtract (II) from (I)

$$ax-by=0$$

which is radical axis of circles

Circle (I) and (II) touch each other

if $CM =$ Radius of (I)

Centre of (I)

$C(-a, 0)$

Also Radius of (I) $\sqrt{g^2+f^2-c}$

$\sqrt{a^2+0-c}$

$\sqrt{a^2-c}$

$CM = \sqrt{a^2 - c}$ where CM is ⊥ distance from c to Redical axis.

$$\frac{|a(-a) - b(0)|}{\sqrt{a^2+b^2}} = \sqrt{a^2 - c}$$

$$\frac{|-a^2|}{\sqrt{a^2+b^2}} = \sqrt{a^2 - c}$$

sq. both side.

$$\frac{a^4}{a^2+b^2} = a^2 - c$$

$$a^4 = a^4 - a^2c + a^2b^2 - b^2c$$

$$(a^2 + b^2)\,c = a^2b^2$$

$$\frac{a^2 + b^2}{a^2b^2} = \frac{1}{c}$$

$$\frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2} = \frac{1}{c}$$

$$\frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{c}$$

Hence Proved.

③ Find the equation of the circle which passes through the points $(2,0)$ & $(0,2)$ and is orthogonal to circle

$$2x^2 + 2y^2 + 5x - 6y + 4 = 0$$

Sol Let eq. of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{—(I)}$$

It passes through $(2,0)$

$$4 + 0 + 4g + 0 + c = 0$$

$$4g + c = -4 \quad \text{—(II)}$$

Also (I) Passes through $(0, 2)$

$$0 + 4 + 0 + 4f + c = 0$$

$$4f + c = -4 \quad \text{— (III)}$$

Subtract (III) from (II)

$$4g + \cancel{c} - 4f - \cancel{c} = -4 + 4$$

$$4g - 4f = 0$$

$$4g = 4f$$

$$g = f. \quad \text{— (IV)}$$

$2x^2 + 2y^2 + 5x - 6y + 4 = 0$ is Orthogonal

to $x^2 + y^2 + 2gx + 2fy + c = 0$

$$x^2 + y^2 + \frac{5}{2}x - \frac{6}{2}y + 2 = 0$$

$$2g\left(\frac{5}{4}\right) + 2f\left(\frac{-6}{4}\right) = c + 2$$

$$5g - 6f = 2c + 4$$

$$5g - 6f = 2(-4 - 4f) + 4 \quad [\text{from (ii)}]$$

$$5g - 6f = -8 - 8f + 4$$

$$5g - 6f + 8f + 4 = 0$$

$$5g + 2f + 4 = 0$$

$$5f + 2f + 4 = 0 \quad [\text{from } (IV)]$$

$$7f + 4 = 0$$

$$f = -4/7.$$

$$g = -4/7.$$

from (II) $\quad c = -4 - 4\left(\frac{-4}{7}\right)$

$$= -4 + \frac{16}{7}$$

$$= \frac{-28+16}{7} = \frac{-12}{7}$$

Put f, g & c in ①

$$x^2+y^2+2\left(\frac{-4}{7}\right)x+2\left(\frac{-4}{7}\right)y+\left(\frac{-12}{7}\right)=0$$

$$7x^2+7y^2-8x-8y-12=0$$

Which is required eq.

Find the eq. of circle which passes through (3,0) touches the y-axis and cuts orthogonally the circle

$$x^2+y^2-6x+4y-3=0$$