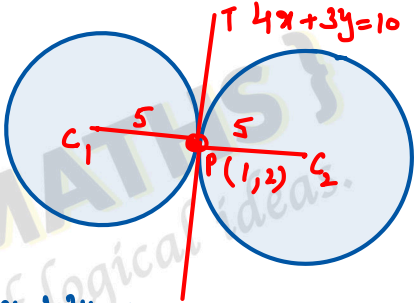
Plane Geometry

Circle

Important questions (PYQ)

Two Circle each of radius 5 units, touch each other at point (1, 2) If the equation of their Common tangent is $4x + 3y = 10$. Find the equation of Circles.

Sol. Two circle Touch each other at P (1,2) also Common tangent $4x+3y=10$



Slope of T $(m_1) = \frac{-4}{3}$ —①

Let C_1 is (x_1, y_1) (C_1 is Centre of Circle)

Slope of C_1P $(m_2) = \frac{y_1-2}{x_1-1}$ —②

Line C_1P & T are $\perp$ to each other

$\therefore \quad m_1 m_2 = -1$

$$\left(\frac{y_1 - 2}{x_1 - 1}\right)\left(\frac{-4}{3}\right) = -1$$

$$4y_1 - 8 = 3x_1 - 3$$

$$3x_1 - 4y_1 = -8 + 3$$

$$3x_1 - 4y_1 = -5$$

$$3x_1 - 4y_1 + 5 = 0 \quad - (ii)$$

C_1P is $\perp$ distance from C_1 to line T

$$C_1P = \frac{|4x_1 + 3y_1 - 10|}{\sqrt{16 + 9}} = 5$$

$$\frac{4x_1 + 3y_1 - 10}{5} = \pm 5$$

either $4x_1 + 3y_1 - 10 = 25$

$$4x_1 + 3y_1 - 35 = 0 \quad \text{— (10)}$$

or $4x_1 + 3y_1 - 10 = -25$

$$4x_4 + 3y_1 + 15 = 0 \quad -(V)$$

Solve (III) and (IV)

$$3x_4 - 4y_1 + 5 = 0 \quad \times 4$$

$$4x_4 + 3y_1 - 35 = 0 \quad \times 3$$

$$12x_4 - 16y_1 + 20 = 0$$

$$-12x_4 + 9y_1 - 105 = 0$$

$$-25y_1 + 125 = 0$$

$$y_1 = \frac{125}{25} = 5$$

Put $y_1 = 5$ in (IV) $\quad x_4 = 5$

$\therefore$ Centre C of Circle is

$(5, 5)$ Radius $= 5$ (Given)

eq. of Circle is

$$(x-5)^2 + (y-5)^2 = 25$$

$$x^2 + 25 - 10x + y^2 + 25 - 10y = 25$$

$$x^2 + y^2 - 10x - 10y + 25 = 0$$

Solve (ii) and (v)

$$3x_1 - 4y_1 + 5 = 0 \quad \times 4$$
$$4x_1 + 3y_1 + 15 = 0 \quad \times 3$$

$$12x_1 - 16y_1 + 20 = 0$$
$$-\ 12x_1 + 9y_1 + 45 = 0$$

$$-25y_1 - 25 = 0$$

$$y_1 = -1$$

Put $y_1 = -1$ in (iii)

$$x_1 = -3.$$

C_2 Centre of other Circle is

$(-3, -1)$ radius = 5 (Given)

$$(x+3)^2 + (y+1)^2 = 25$$

$$x^2 + 9 + 6x + y^2 + 1 + 2y = 25$$

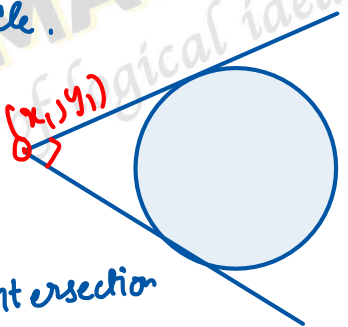
$$x^2 + y^2 + 6x + 2y + 10 - 25 = 0$$

$$x^2 + y^2 + 6x + 2y - 15 = 0$$

(2) Find the locus of the point of intersection of two perpendicular tangents to circle.

Sol. Let eq of Circle

$x^2 + y^2 = a^2$



Now Let Point of intersection of tangents is $P(x_1, y_1)$

eq. of tangent to the circle.

$$y = mx + a\sqrt{1+m^2}$$

It passes through (x_1, y_1)

$$y_1 = mx_1 + a\sqrt{1+m^2}$$

$$y_1 - mx_1 = a\sqrt{1+m^2}$$

sq. both sides.

$$(y_1 - mx_1)^2 = a^2(1+m^2)$$

$$y_1^2 + m^2 x_1^2 - 2mx_1y_1 = a^2(1+m^2)$$

$$y_1^2 + m^2 x_1^2 - 2mx_1y_1 - a^2 - a^2m^2 = 0$$

$$m^2(x_1^2 - a^2) - 2mx_1y_1 + y_1^2 - a^2 = 0 \quad \text{--- (1)}$$

Which is quad. in m.

Let m_1, m_2 be the roots of eq. (1)

Tangents are $\perp$ to each other (Given)

$\therefore \quad m_1 m_2 = -1$

from (1) $m_1 m_2 = \dfrac{y_1^2 - a^2}{x_1^2 - a^2} = -1$

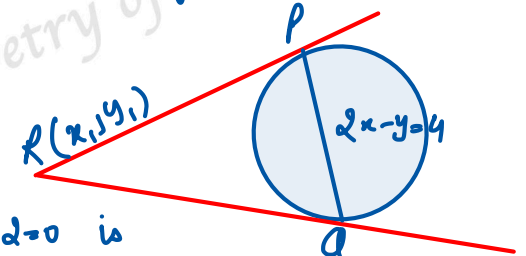
$$y_1^2 - a^2 = -x_1^2 + a^2$$

$$y_1^2 + x_1^2 = 2a^2$$

$\therefore$ Locus of (x_1, y_1) is $x^2 + y^2 = 2a^2$

which is eq of circle.

③ The line $2x-y=4$ meets the circle $x^2+y^2-6x+2y+2=0$ at the points P and Q. If the tangents of P and Q meet at R. find the coordinates of R.



Sol.

$x^2+y^2-6x+2y+2=0$ is eq. of given circle

Compare with $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \quad 2f = 2 \quad c = 2.$

$g = -3 \quad f = 1$

PQ is Chord of Contact where $R(x_1, y_1)$

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$xx_1 + yy_1 - 3x - 3x_1 + y + y_1 + 2 = 0$

$(x_1 - 3)x + (y_1 + 1)y - 3x_1 + y_1 + 2 = 0 \quad -①$

Also PQ is $2x - y = 4$ (Given) $\quad -②$

$\therefore$ (i) and (ii) represent same line.

$$\frac{2}{x_1 - 3} = \frac{-1}{y_1 + 1} = \frac{4}{3x_1 - y_1 - 2}$$

$$2y_1 + 2 = -x_1 + 3$$

$$x_1 + 2y_1 - 1 = 0 \quad - (iii)$$

$$-3x_1 + y_1 + 2 = 4y_1 + 4$$

$$-3x_1 - 3y_1 - 2 = 0$$

$$3x_1 + 3y_1 + 2 = 0 \quad - (iv)$$

Solve (iii) & (iv)

$$x_1 + 2y_1 - 1 = 0 \quad \times 3$$
$$3x_1 + 3y_1 + 2 = 0 \quad \times 1$$

$$3x_1 + 6y_1 - 3 = 0$$
$$- \; 3x_1 \pm 3y_1 \pm 2 = 0$$
$$3y_1 = 5$$
$$y_1 = \frac{5}{3}$$

Put y_1 in (iii)

$$x_1 = -7/3.$$

$$\therefore \quad R(x_1, y_1) = \left(-7/3, 5/3\right)$$