Plane Geometry

Circle

Full Chapter Revision

* Central form of eq. of Circle.

Centre C (h, k)

& Radius r the

eq. is Circle is $(x-h)^2 + (y-k)^2 = r^2$

* Standard form of eq. of Circle

Centre (0, 0) Radius r

eq of Circle is

$$x^2 + y^2 = r^2$$

* General form of eq of Circle is

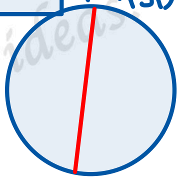
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $(-g, -f)$

Radius $\sqrt{g^2 + f^2 - c}$

✲ Diameter form of eq. of circle

$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$$



where (x_1, y_1) & (x_2, y_2) are end points of diameter of circle.

✲ Intercept form of circle

eq. of circle passing through origin and making intercep a and b on axis is

$$x^2 + y^2 - ax - by = 0$$

Parametric eq. of Circle

$$\left.\begin{aligned} x &= h + r\cos\theta \\ y &= k + r\sin\theta \end{aligned}\right]$$ are parametric eq. of $(x-h)^2 + (y-k)^2 = r^2$

Eq. of Circle passing through intersection of Circles

$$S = x^2 + y^2 + 2gx + 2fy + c = 0 \quad -(1)$$

$$S' = x^2 + y^2 + 2g'x + 2f'y + c' = 0 \quad -(11)$$

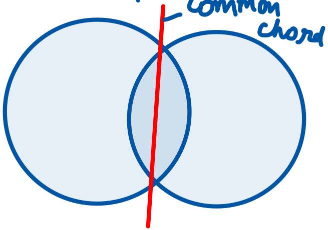
is $\boxed{S + kS' = 0}$ $\quad k \neq -1$

If $k = -1$ $\quad S - S' = 0$

$$\boxed{2(g - g')x + 2(f - f')y + c_1 - c_2 = 0}$$

which is the straight line passing through point of intersection of circles ① & ②

i.e. Common chord of circles.



✪ eq. of Circle passing through intersection of Circle

$$S = x^2 + y^2 + 2gx + 2fy + c = 0$$

and a line $L = lx + my + n = 0$ is

$$S + KL = 0$$

✪ eq. of any Circle through the intersection of Circle $S = 0$ & $S' = 0$ is

$$S + k(S - S') = 0$$

* eq. of Tangent to Circle at (x_1, y_1)

$$S_1 = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

* eq of Normal at (x_1, y_1) of Circle

$x^2 + y^2 = a^2$ is

$$xy_1 - x_1y = 0$$

Condition of Tangency

line $lx + my + n = 0$ is tangent to $x^2 + y^2 = a^2$ if

$$a^2(l^2 + m^2) = n^2$$

Point of Contact $\left(-\frac{a^2 l}{n}, -\frac{a^2 m}{n}\right)$

* Line $y = mx + c$ is tangent to Circle $x^2 + y^2 = a^2$ if

$$c^2 = a^2(1 + m^2)$$

Point of Contact $\left(\pm \frac{am}{\sqrt{1+m^2}}, \mp \frac{a}{\sqrt{1+m^2}} \right)$

Chord of Contact

eq. of Chord of Contact of tangents drawn from (x_1, y_1) is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

same as Tangent.

eq. of Polar of (x_1, y_1) is

$$\boxed{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0}$$

* Pole of line $lx + my + n = 0$ w.r.t. $x^2 + y^2 = a^2$ is

$$\left(-\frac{a^2 l}{n}, -\frac{a^2 m}{n}\right)$$

* Orthogonal Circles:- Two circles are said to intersect Orthogonally if angle between them is right angle.

* Angle of intersection of two Circles.

$$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$$

r_1, r_2 are radii of Circles & d is distance between Centres

* <u>Condition of Orthogonality</u>.

r_1, r_2 are radii of Circles
d is distance between Centres.

$$\boxed{r_1^2 + r_2^2 = d^2}$$

Condition of Orthogonality of Circles.

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \text{ is}$$

$$\boxed{2g_1g_2 + 2f_1f_2 = c_1 + c_2}$$

Length of tangent from exterior point $P(x_1, y_1)$ is

$$\boxed{\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}}$$