EIGEN VALUES AND CAYLEY-HAMILTON THEOREM

Important Question (PYQ)

Prove that characteristic roots of a hermitian matrix are real.

[Characteristic Roots/ eigen values / Proper values/ latent roots/ spectral values].

Proof

Let A is hermitian matrix

$A^{\theta} = A$ —— (1)

Let λ be the eigen value of A

$\therefore \exists$ a non zero Column matrix $n \times 1$ s.t.

$$AX = \lambda X \quad \text{--- (2)}$$

$$X^{\theta}(AX) = X^{\theta}(\lambda X)$$

$$X^{\theta}AX = \lambda (X^{\theta}X)$$

$$(X^{\theta}AX)^{\theta} = (\lambda(X^{\theta}X))^{\theta}$$

$$x^{\theta} A^{\theta} (x^{\theta})^{\theta} = \bar{\lambda} \left(x^{\theta} (x^{\theta})^{\theta} \right)$$

[from ①]

$$x^{\theta} A x = \bar{\lambda} \left(x^{\theta} x \right)$$

$$x^{\theta} \lambda x = \bar{\lambda} \left(x^{\theta} x \right)$$

[from ②]

$$\lambda (x^{\theta} x) - \bar{\lambda} (x^{\theta} x) = 0$$

$$(\lambda - \bar{\lambda})(x^{\theta} x) = 0$$

$$\lambda - \bar{\lambda} = 0$$

[$x^{\theta} \neq 0$ $x \neq 0$]

$\lambda = \bar{\lambda} \Rightarrow \lambda$ is <u>real</u>. <u>Hence proved.</u>