

# EIGEN VALUES AND CAYLEY-HAMILTON THEOREM

## Important Question (PYQ)

Prove that characteristic roots of a hermitian matrix are real

[Characteristic Roots / eigen values / Proper values / latent roots / spectral values].

Proof

Let  $A$  is hermitian matrix

$$A^0 = A \quad \text{--- (1)}$$

Let  $\lambda$  be the eigen value of  $A$

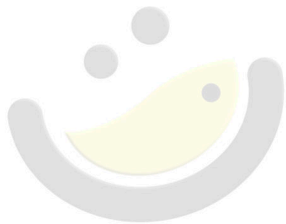
$\therefore \exists$  a non zero Column matrix  
 $n \times 1$  s.t.

$$AX = \lambda X \quad \text{--- (1)}$$

$$X^{\theta}(AX) = X^{\theta}(\lambda X)$$

$$X^{\theta}AX = \lambda(X^{\theta}X)$$

$$(X^{\theta}AX)^{\theta} = (\lambda(X^{\theta}X))^{\theta}$$



$$x^0 A^0 (x^0)^0 = \bar{\lambda} (x^0 (x^0)^0)$$

$$x^0 A x = \bar{\lambda} (x^0 x) \quad [\text{from ①}]$$

$$x^0 \lambda x = \bar{\lambda} (x^0 x) \quad [\text{from ②}]$$

$$\lambda (x^0 x) - \bar{\lambda} (x^0 x) = 0$$

$$(\lambda - \bar{\lambda})(x^0 x) = 0$$

$$\lambda - \bar{\lambda} = 0$$

$\lambda = \bar{\lambda} \Rightarrow \lambda$  is real Hence proved.

