Plane Geometry

Transformation of axes in 2D

IMPORTANT QUESTIONS

Transform to axis inclined at angle $\tan^{-1} 2$, to the original axes of the equation $11x^2 - 4xy + 14y^2 = 5$

Sol.

transformed equation is

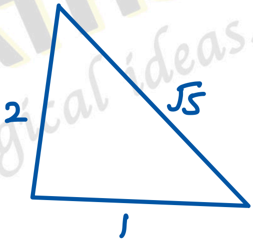
$$11x^2 - 4xy + 14y^2 = 5$$

$$11x'^2 - 4x'y' + 14y'^2 = 5 \quad \text{—} \circledast$$

Now

$$\theta = \tan^{-1} 2$$

$$\tan\theta = \frac{2}{1}$$



$P = 2$

$B = 1$

P B P
H H B

$$\sin\theta = \frac{2}{\sqrt{5}} \qquad \cos\theta = \frac{1}{\sqrt{5}}$$

$$\left.\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}\right\} \quad (1)$$

Put values of $\sin\theta$ and $\cos\theta$ in (1)

$$x' = \frac{x}{\sqrt{5}} + \frac{y \times 2}{\sqrt{5}} = \frac{x+2y}{\sqrt{5}}$$

$$y' = -\frac{x \cdot 2}{\sqrt{5}} + y\left(\frac{1}{\sqrt{5}}\right) = \frac{-2x+y}{\sqrt{5}}$$

Put x' and y' in A

$$11x'^2 - 4x'y' + 14y'^2 = 5$$

$$11\left(\frac{x+2y}{\sqrt{5}}\right)^2 - 4\left(\frac{x+2y}{\sqrt{5}}\right)\left(\frac{-2x+y}{\sqrt{5}}\right) + 14\left(\frac{-2x+y}{\sqrt{5}}\right)^2 = 5$$

$$11(x^2+4y^2+4xy) - 4(-2x^2+xy-4xy+2y^2) + 14(4x^2+y^2-4xy) = 25$$

$$11x^2 + 44y^2 + 44xy + 8x^2 - 4xy + 16xy - 8y^2$$

$$+ 56x^2 + 14y^2 - 56xy = 25$$

$$75x^2 + 50y^2 = 25$$

$$3x^2 + 2y^2 = 1$$

Which is the required equation

By the suitable transformation remove terms involving x, y from the equation

$$y^2 - 2xy + 2x^2 + 2x - 2y = 0 \quad \text{---(1)}$$

Sol. To remove x, y terms shift origin to (h, k)

$$x = x' + h$$

$$y = y' + k$$

Put values of x and y in ①

$$(y'+k)^2 - 2(x'+h)(y'+k) + 2(x'+h)^2 +$$
$$2(x'+h) - 2(y'+k) = 0$$

$$y'^2 + k^2 + 2y'k - 2(x'y' + x'k + y'h + hk)$$
$$+ 2(x'^2 + h^2 + 2x'h) + 2x' + 2h - 2y'$$
$$- 2k = 0$$

$$2x'^2 + y'^2 - 2x'y' + x'(-2k + 4h + 2) + y'(2k - 2h - 2) + k^2 - 2hk + 2h^2 + 2h - 2k = 0 \quad \text{--- (11)}$$

$$-2k + 4h + 2 = 0$$

$$2k - 2h - 2 = 0$$

$$2h = 0$$

$$h = 0$$

$$k = 1$$

$$h = 0 \qquad k = 1.$$

Put $h=0$ and $k=1$ in (II)

$$2x'^2 + y'^2 - 2x'y' - 1 = 0$$

$$2x'^2 + y'^2 - 2x'y' = 1$$

Change x' to x and y' to y.

$$2x^2 + y^2 - 2xy = 1$$