

CALCULUS

State and Prove Taylor's Theorem

Statement :- If a function f defined in $[a, b]$

s.t.
(i) $f, f', f'', f''' \dots f^{n-1}(x)$ are
continuous in $[a, b]$

(ii) f^n exists in (a, b)

then \exists at least one real no. $c \in (a, b)$
s.t.

$$\begin{aligned}
 f(b) = & f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!} f''(a) \\
 & + \dots + \frac{(b-a)^{n-1}}{(n-1)!} f^{(n-1)}(a) \\
 & + \frac{(b-a)^n}{n!} f^n(c)
 \end{aligned}$$

Sol.

Consider a function ϕ s.t

$$\phi(x) = f(x) + (b-x)f'(x) + \frac{(b-x)^2}{2!} f''(x) +$$

$$\dots + \frac{(b-x)^{n-1}}{(n-1)!} f^{n-1}(x) +$$

$$\frac{(b-x)^n}{n!} A. \quad \text{--- ①}$$

Where A is constant to determine s.t

$$\phi(a) = \phi(b)$$

$$f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!} f''(a) + \dots$$

$$\dots + \frac{(b-a)^{n-1}}{(n-1)!} f^{n-1}(a) + \frac{(b-a)^n}{n!} A = f(b) \quad \text{--- ②}$$

$f, f'', f', f'''' \dots f^{n-1}$ are

Continuous in $[a, b]$

Sum of Continuous functions is Continuous

$\therefore \phi(x)$ is Continuous in $[a, b]$ - (3)

lly $\phi(x)$ is derivable in (a, b) - (4)

$$\phi(a) = \phi(b) \quad - (5)$$

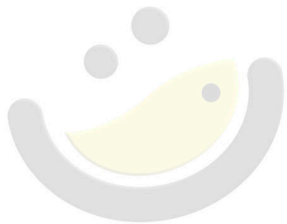
from (3), (4) and (5)

By Rolle's thm.

$\exists c \in (a, b)$ s.t.

$$\phi'(c) = 0$$

$$\phi(x) = f(x) + (b-x)f'(x) + \frac{(b-x)^2}{2!}f''(x) + \dots + \frac{(b-x)^{n-1}}{(n-1)!}f^{(n-1)}(x) + \frac{(b-x)^n}{n!}A$$



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$$Q'(x) = f'(x) + \left[(b-x)f''(x) + f'(x)(-1) \right] +$$

$$\left[\frac{(b-x)^2}{2!} f'''(x) + f''(x) \cdot \frac{2(b-x)(-1)}{2!} \right]$$

$$+ \dots + \left[\frac{(b-x)^{n-1}}{(n-1)!} f^{(n)}(x) + f^{(n-1)}(x) \cdot \frac{(n-1)(b-x)^{n-2}(-1)}{(n-1)!} \right]$$

$$+ \frac{n(b-x)^{n-1}}{n!} (-1) A$$



$$= \frac{(b-x)^{n-1}}{(n-1)!} f^n(x) - \frac{(b-x)^{n-1}}{(n-1)!} A$$

$$\underline{Q'(x)} = \frac{(b-x)^{n-1}}{(n-1)!} (f^n(x) - A)$$

from 6

$$Q'(c) = 0$$

$$\frac{(b-c)^{n-1}}{(n-1)!} [f^n(c) - A] = 0$$

$$f^n(c) - A = 0$$

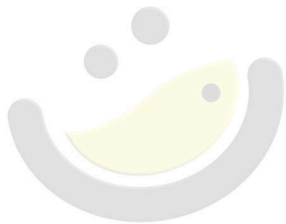
$$f^n(c) = A$$

Put $f^n(c) = A$ in ②

$$f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \dots$$

$$\dots \frac{(b-a)^{n-1}}{(n-1)!} f^{(n-1)}(a) + \frac{(b-a)^n}{n!} f^{(n)}(c) = f(b)$$

Hence Proved.



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