CALCULUS

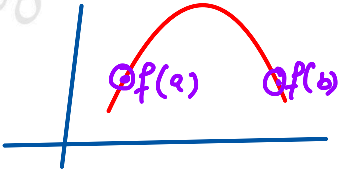
Rolle's Theorem: State and Proof

Statement:- If f is a function

(i) Continous in closed interval $[a,b]$

(ii) derivable in (a,b)

(iii) $f(a) = f(b)$



then $\exists$ at least one real no. s.t

$f'(c) = 0 \quad c \in (a,b)$

Sol. f is Continous on $[a,b]$

$\Rightarrow$ f is Bounded and attains its g.l.b and l.u.b

Let l.u.b $= M$

g.l.b $= m$.

Two Cases arise

$m = M$ and $m \neq M$.

Case I

$m = M$

$\Rightarrow$ f is a constant function

Let $f(x) = d$. [d is real No.]

$\Rightarrow f'(x) = 0$

Hence Proved.

Case II $m \neq M$

Since $f(a) = f(b)$ [Given]

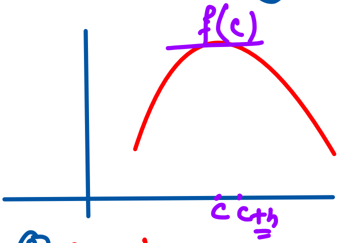
$\Rightarrow$ f has at least one value diff from $f(a)$ and $f(b)$

Let $f(c) = M$ where $c \in (a,b)$

f is differentiable at (a,b) [Given]

$\therefore f'(c)$ exist

$\therefore f'(c) = \dfrac{f(c+h) - f(c)}{h}$ will exist — ①



Now

$f(c) = M$

$f(c+h) \leq f(c)$

$f(c+h) - f(c) \leq 0$ — ② (-ve)

OMG{MATHS} The poetry of logical ideas.

When $h > 0$

$$f'(c) \leq 0 \quad \text{——} ③ \quad [\text{from ① and ②}]$$

When $h < 0$

$$f'(c) \geq 0 \quad \text{——} ④ \quad [\text{from ① and ②}]$$

from ③ and ④

$$f'(c) = 0$$

Hence Proved