

Leibnitz's Theorem: Example

Most Important

$$y = (\sin^{-1} x)^2 \quad \text{find } y_n(0)$$

So

$$y = (\sin^{-1} x)^2$$

$$y_1 = 2 \sin^{-1} x \frac{d}{dx} \sin^{-1} x$$

$$= 2 \sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}}$$



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$$\sqrt{1-x^2} y_1 = 2 \sin^{-1}x$$

Squaring both sides

$$(1-x^2)(y_1)^2 = 4(\sin^{-1}x)^2$$

$$(1-x^2)y_1^2 = 4y_1$$

$$(1-x^2)y_1^2 - 4y_1 = 0$$

Diff again.

$$(1-x^2) 2 y_1 \cdot y_2 + y_1^2 \cdot (-2x) - 4y_1 = 0$$

$$2y_1 \left[(1-x^2)y_2 + (-xy_1) - 2 \right] = 0$$

$$(1-x^2)y_2 - xy_1 - 2 = 0 \quad \text{--- } \textcircled{A}$$

Diff n times Both sides

$$\left((1-x^2)y_2 \right)_n - (xy_1)_n - (2)_n = 0$$

$$\underline{\left((1-x^2)y_2 \right)_n} - \underline{(xy_1)_n} = 0 \quad \text{--- } \textcircled{1}$$

$$\left((1-x^2)y_2 \right)_n$$

\downarrow \downarrow
 u v

$$(uv)_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots + {}^nC_r u_{n-r} v_r + \dots + {}^nC_n u v_n$$

$$= {}^nC_0 y_{n+2} (1-x^2) + {}^nC_1 y_{n+1} (-2x) + {}^nC_2 y_n (-2)$$

$$= (1-x^2) y_{n+2} + n (-2x) y_{n+1} + \frac{n(n-1)}{2} (-2) y_n$$

$$= (1-x^2) y_{n+2} - 2nx y_{n+1} - n(n-1) y_n \quad \text{--- (i)}$$

$$\binom{x y_1}{u \quad u}_n$$

$$(uv)_n = {}^n C_0 u_n v + {}^n C_1 u_{n-1} v_1 + {}^n C_2 u_{n-2} v_2 + \dots + {}^n C_r u_{n-r} v_r + \dots + {}^n C_n u v_n$$

$$= {}^n C_0 y_{n+1} x + {}^n C_1 y_n (1)$$

$$= x y_{n+1} + n y_n \quad \text{--- (ii)}$$

Put (ii) and (iii) in (i)

$$(1-x^2)y_{n+2} - 2nx y_{n+1} - n(n-1)y_n - x y_{n+1} - n y_n = 0$$

$$(1-x^2)y_{n+2} - (2n+1)y_{n+1}x - (n^2-n+n)y_n = 0$$

$$(1-x^2)y_{n+2} - (2n+1)y_{n+1}x - n^2y_n = 0 \quad \text{---IV}$$

y at $x=0$

$$y = 0 \quad \text{--- (i)}$$

$$y_1 = 0 \quad \text{--- (ii)}$$

from \star $y_2 = 2 \quad \text{--- (iii)}$

Now from IV $y_{n+2} = n^2 y_n(0) \quad \text{---(iv)}$

from (iv) $y_3 = 1(0)$ — from (ii)

$$y_3 = 0$$

$$y_4 = 2^2 y_2(0) = 2^2 \cdot 2 \text{ — from (iii)}$$

$$y_5 = 3^2 y_3(0) = 0$$

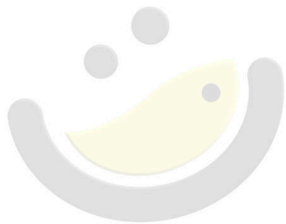
$$y_6 = 4^2 y_4(0) = 4^2 \cdot 2^2 \cdot 2$$

⋮

$$f(n \in \mathbb{O}) = \begin{cases} 0 & n \text{ is odd} \\ 2 \cdot 2^2 \cdot 4^2 \dots (n-2)^2 & \text{when } n \text{ is even} \\ & \text{and } n \neq 2. \end{cases}$$

Ans

like



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