Leibnitz's Theorem: Example

Most Important

$$y = a\cos(\log x) + b\sin(\log x)$$

Show that

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2+1)y_n = 0$$

$$y = a\cos(\log x) + b\sin(\log x)$$

$$y_1 = a\left(-\sin(\log x)\right) 1/x + b\cos(\log x)\cdot\frac{1}{x}$$

$$xy_1 = -a\sin(\log x) + b\cos(\log x)$$

$$xy_2 + y_1(1) = -a\cos(\log x)\cdot\frac{1}{x} + b\left(-\sin(\log x)\right)\frac{1}{x}$$

$$= -\frac{1}{x}\left[a\cos(\log x) + b\sin(\log x)\right]$$

$$x^2 y_2 + xy_1 = -y$$

$$x^2 y_2 + xy_1 + y = 0$$

$$(x^2 y_2)_n + (xy_1)_n + y_n = 0 \quad \text{--- (1)}$$

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$$\left(\underset{\substack{\downarrow\\ v}}{x^2}\;\underset{\substack{\downarrow\\ u}}{y_2}\right)_n$$

$$(uv)_n = {}^nC_0\, u_n v + {}^nC_1\, u_{n-1}v_1 + {}^nC_2\, u_{n-2}v_2 + \dots\dots\dots + {}^nC_r\, u_{n-r}v_r + \dots\dots\dots + {}^nC_n\, u\, v_n$$

$$= {}^nC_0\, y_{n+2} \cdot x^2 + {}^nC_1\, y_{n+1}(2x) + {}^nC_2\, y_n(2)$$

$$= y_{n+2}\,x^2 + 2nx\, y_{n+1} + n(n-1)\, y_n \quad \text{— (ii)}$$

$$(uv)_n = {}^nC_0\, u_n v + {}^nC_1\, u_{n-1}v_1 + {}^nC_2\, u_{n-2}v_2 + \dots\dots\dots + {}^nC_r\, u_{n-r}v_r + \dots\dots\dots + {}^nC_n\, u\, v_n$$

$$\left(\underset{\substack{\downarrow\\ v}}{x}\,\underset{\substack{\downarrow\\ u}}{y_1}\right)_n = {}^nC_0\, y_{n+1}\, x + {}^nC_1\, y_n = x\,y_{n+1} + n\,y_n \quad \text{— (iii)}$$

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Put (ii) and (iii) in (i)

$$x^2 y_{n+2} + 2nx\, y_{n+1} + n(n-1) y_n + x y_{n+1} + n y_n + y_n = 0$$

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n(n-1) + n + 1) y_n = 0$$

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - \cancel{n} + \cancel{n} + 1) y_n = 0$$

Hence Proved

$$\boxed{x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2+1) y_n = 0}$$

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