

# CALCULUS

## State and Prove Lagrange's Mean Value Theorem

Statement :- If a function  $f$  is

(i) Continuous in  $[a, b]$

(ii) differentiable in  $(a, b)$

then  $\exists$  at least one real no.

$c \in (a, b)$  s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Proof ∴

Consider a function

$$\phi(x) = f(x) + Ax \quad \text{--- (1)}$$

where  $A$  is constant to determine

s.t.  $\phi(a) = \phi(b)$

from (1)

$$f(a) + A \cdot a = f(b) + A \cdot b$$

$$A \cdot (a - b) = f(b) - f(a)$$

$$-A (b - a) = f(b) - f(a)$$

$$-A = \frac{f(b) - f(a)}{b - a} \quad \text{--- (2)}$$

(i)  $f$  is Continuous in  $[a, b]$  [Given]  
 $Ax$  is Continuous in  $[a, b]$  [ $\because Ax$  is a polynomial]

also sum of Continuous functions is Continuous.

$\therefore f(x)$  is Continuous in  $[a, b]$  {from ①}  
- ③

(ii)  $f$  is differentiable in  $(a, b)$  [Given]  
 $Ax$  is differentiable in  $(a, b)$  ( $Ax$  is polynomial)

Sum of differentiable functions is differentiable.

$\therefore \phi(x)$  is differentiable in  $(a, b)$  [from ①]  
- ④

Now

$$\phi(a) = \phi(b) \quad \text{- ⑤}$$

$\therefore$  By Rolle's thm. {from ③, ④ & ⑤}

$$\phi'(c) = 0$$

$$f'(c) + A = 0$$

$$\phi(x) = f(x) + Ax$$

$$\phi'(x) = f'(x) + A$$

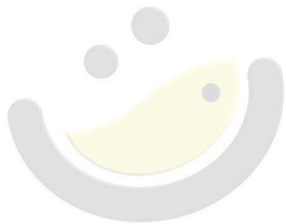
$$f'(c) = -A$$

$$= \frac{f(b) - f(a)}{b - a} \quad \{ \text{from } \textcircled{2} \}$$

Hence

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Hence Proved.



ONG { MATHS }  
The poetry of logical ideas.