Calculus

L'Hospital Rule

Evaluate

$$\lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$$

Sol.

$$\frac{e^0 - e^0 - 2\log 1}{0} = \frac{0}{0} \text{ form.}$$

By L' Hospital Rule.

$$\lim_{x \to 0} \frac{f'(x)}{g'(x)} = \lim_{x \to 0} \frac{e^x + e^{-x} - \frac{2}{1+x}}{x \cos x + \sin x} \quad \left(\frac{0}{0}\right) \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x - e^{-x} + \frac{2}{(1+x)^2}}{x(-\sin x) + \cos x + \cos x}$$

$$= \frac{e^0 - e^0 + \frac{2}{1}}{0 + 1 + 1} = \frac{2}{2} = 1 \quad \text{Ans}$$