Calculus

L'Hospital Rule

find the Values of a and b so that $\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3}$ exists and is equal to 1.

Sol. $\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = \left(\frac{0}{0}\right)$ form

By L' hospital Rule

$$\lim_{x \to 0} \frac{x(-a\sin x) + (1 + a\cos x) - b\cos x}{3x^2} \quad \text{--- (1)}$$

Now Denominator of (1) is zero at $x = 0$

$\therefore$ The given may exist only when numerator of (1) is zero at $x = 0$

$$1 + a - b = 0$$

$$a = b - 1 \quad \text{--- (2)}$$

from (1)

$$\lim_{x\to 0} \frac{x(-a\cos x) + (-a\sin x) + (-a\sin x) + b\sin x}{6x} \quad \left(\frac{0}{0}\right) \text{ form}$$

$$\lim_{x\to 0} \frac{x(+a\sin x) + (-a\cos x) + (-a\cos x) - a\cos x + b\cos x}{6}$$

$$= \frac{b - 3a}{6} = 1 \qquad \left[\because \lim_{x\to 0} \frac{f(x)}{g(x)} = 1 \text{ given}\right]$$

$$b - 3a = 6$$

$$b - 3(b - 1) = 6 \quad \text{from } ②$$

$$b - 3b + 3 = 6$$

$$-2b = 6 - 3$$

$$b = -3/2$$

$$a = b - 1$$

$$= \frac{-3}{2} - 1 = \frac{-3 - 2}{2} = -5/2 \text{ Ans}$$