Calculus

L'Hospital Rule

Prove that

$$\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x} = 2$$

Sol

$$\frac{1^1 - 1}{1 - 1 - \log 1} = \frac{0}{0} \text{ form}$$

By. L' Hospital Rule

$$\lim_{x\to 1} \frac{f'(x)}{g'(x)}$$

$$= \lim_{x\to 1} \frac{x^x(1+\log x) - 1}{1 - \frac{1}{x}} \quad \left(\frac{0}{0}\right) \text{ form}$$

$$= \lim_{x\to 1} \frac{x^x \cdot \frac{1}{x} + (1+\log x)\, x^x (1+\log x)}{1/x^2}$$

$$y = x^x$$

$$\log y = x \log x$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \log x$$

$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\frac{dy}{dx} = y(1+\log x)$$

$$= x^x(1+\log x)$$

$$= \frac{1' \cdot 1 + (1+\log 1)\, y' (1+\log 1)}{\frac{1}{1}}$$

$$= \frac{1 + 1\,(1)(1)}{1} = 1+1 = 2$$

Ans

L.H.S = R.H.S

Hence Proved

$y' = x^x (1+\log x)$

$$\frac{dy'}{dx'} = x^x\left(\frac{1}{x}\right) + (1+\log x)$$

$x^x(1+\log x)$