$a = 3 \qquad b = 3$

$h = 1.$

$\because \; a = b = 3$

To remove term containing $x'y'$ we rotate the axes through an angle $\theta = 45^\circ$

$\sin\theta = \frac{1}{\sqrt{2}} \qquad \cos\theta = \frac{1}{\sqrt{2}}$

$$x' = X\cos\theta - Y\sin\theta$$

$$y' = X\sin\theta + Y\cos\theta.$$

$$x' = X\frac{1}{\sqrt{2}} - Y\frac{1}{\sqrt{2}} = \frac{X-Y}{\sqrt{2}}$$

$$y' = X\frac{1}{\sqrt{2}} + Y\frac{1}{\sqrt{2}} = \frac{X+Y}{\sqrt{2}}$$

Put x' and y' in 3

$$3\left(\frac{x-y}{\sqrt{2}}\right)^2 + 3\left(\frac{x+y}{\sqrt{2}}\right)^2 + 2\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) - 1 = 0$$

$$\frac{3(x^2+y^2-2xy)}{2} + \frac{3(x^2+y^2+2xy)}{2} + \frac{2(x^2-y^2)}{2} = 1$$

$$3x^2 + 3y^2 - 6xy + 3x^2 + 3y^2 + 6xy + 2x^2 - 2y^2 = 2$$

$$8x^2 + 4y^2 = 2$$

$$4x^2 + 2y^2 = 1$$

Change X to x and Y to y.

$$4x^2 + 2y^2 = 1.$$

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(2) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a'x^2 + 2h'xy + b'y^2 = 1$ represent the same conic and the axes are rectangular then

$$(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$$

Sol.

$$ax^2 + 2hxy + by^2 = 1 \quad - (1)$$

and $$a'x^2 + 2h'xy + b'y^2 = 1 \quad - (2)$$

Now (2) is transformed form of (1)

trace

$$a + b = a' + b' \quad - (3)$$

$$h^2 - ab = h'^2 - a'b' \quad - (4)$$

Squaring (3)

$$(a+b)^2 = (a'+b')^2$$

$$a^2 + b^2 + 2ab = a'^2 + b'^2 + 2a'b' \quad \text{--- (5)}$$

Multiply equation 4 by (4).

$$4h^2 - 4ab = 4h'^2 - 4a'b' \quad \text{--- (6)}$$

Add (5) & (6)

$$a^2 + b^2 + 2ab + 4h^2 - 4ab$$
$$= a'^2 + b'^2 + 2a'b' + 4h'^2 - 4a'b'$$

$$a^2 + b^2 - 2ab + 4h^2 = a'^2 + b'^2 - 2a'b' + 4h'^2$$

$$(a-b)^2 + 4h^2 = (a'-b')^2 + 4h'^2$$

Hence Proved.