

## CALCULUS : Successive Differentiation

$$p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$$

Prove that  $p + \frac{d^2 p}{d\theta^2} = \frac{a^2 b^2}{p^3}$

Sol

$$p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$$

$$= a^2 \left( \frac{1 + \cos 2\theta}{2} \right) + b^2 \left( \frac{1 - \cos 2\theta}{2} \right)$$



$$2p^2 = a^2 + a^2 \cos 2\theta + b^2 - b^2 \cos 2\theta$$

$$= (a^2 + b^2) + (a^2 - b^2) \cos 2\theta$$

$$2p^2 - (a^2 + b^2) = (a^2 - b^2) \cos 2\theta \quad \text{--- (1)}$$

Differentiate both sides

$$2 \left( 2p \frac{dp}{d\theta} \right) = (a^2 - b^2) (-\sin 2\theta) (2)$$

$$2p \frac{dp}{d\theta} = -(a^2 - b^2) \sin 2\theta \quad \text{--- (11)}$$

Squaring and adding (1) and (11)

$$4p^4 + (a^2 + b^2)^2 - 2 \cdot 2p^2(a^2 + b^2) + 4p^2 \left(\frac{dp}{d\theta}\right)^2 \\ = (a^2 - b^2)^2 \cos^2 2\theta + (a^2 - b^2)^2 \sin^2 2\theta$$

$$4p^4 + (a^2 + b^2)^2 - 4p^2(a^2 + b^2) + 4p^2 \left(\frac{dp}{d\theta}\right)^2 \\ = (a^2 - b^2)^2 (\cos^2 2\theta + \sin^2 2\theta)$$

$$4p^4 + (a^2 + b^2)^2 - 4p^2(a^2 + b^2) + 4p^2 \left(\frac{dp}{do}\right)^2$$

$$- (a^2 - b^2)^2 = 0$$

$$\Rightarrow 4p^4 + 4p^2 \left(\frac{dp}{do}\right)^2 - 4p^2(a^2 + b^2) + \cancel{a^4} + \cancel{b^4} + 2a^2b^2 - \cancel{a^4} - \cancel{b^4} + 2a^2b^2 = 0$$

$$\Rightarrow 4p^4 + 4p^2 \left(\frac{dp}{do}\right)^2 - 4p^2(a^2 + b^2) + 4a^2b^2 = 0$$

$$\Rightarrow 4P^2 \left( P^2 + \left( \frac{dP}{d\theta} \right)^2 - (a^2 + b^2) + \frac{a^2 b^2}{P^2} \right) = 0$$

$$\Rightarrow P^2 + \left( \frac{dP}{d\theta} \right)^2 - (a^2 + b^2) + (a^2 b^2) P^{-2} = 0$$

Differentiate Both side

$$\Rightarrow 2P \frac{dP}{d\theta} + 2 \left( \frac{dP}{d\theta} \right)^{2-1} \frac{d}{d\theta} \cdot \frac{dP}{d\theta} + a^2 b^2 (-2) P^{-2-1} \frac{dP}{d\theta} = 0$$

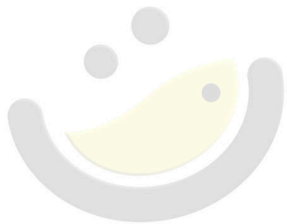
$$\Rightarrow 2p \frac{dp}{d\theta} + 2 \frac{dp}{d\theta} \cdot \frac{d^2p}{d\theta^2} - \frac{2a^2b^2}{p^3} \frac{dp}{d\theta} = 0$$

$$\Rightarrow 2 \frac{dp}{d\theta} \left[ p + \frac{d^2p}{d\theta^2} - \frac{a^2b^2}{p^3} \right] = 0$$

$$\Rightarrow p + \frac{d^2p}{d\theta^2} - \frac{a^2b^2}{p^3} = 0$$

$$\Rightarrow \rho + \frac{d^2 \rho}{d\theta^2} = \frac{a^2 b^2}{\rho^3}$$

Hence proved.



OMG! MATHS }  
The poetry of logical ideas.