1. $x = a(\cos t + t \sin t)$

$y = a(\sin t - t \cos t)$

Compute $\frac{d^2y}{dx^2}$ at $t = \pi/4$

---

2. $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Find the value of $\frac{d^2y}{dx^2}$ at $x = a$.

$x = a(\cos t + t \sin t)$

$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t)$

$\frac{dx}{dt} = at\cos t$ —(I)

$y = a(\sin t - t\cos t)$

$\frac{dy}{dt} = a[\cos t - (t(-\sin t) + \cos t)]$

$= a\{\cos t + t\sin t - \cos t\}$

$\frac{dy}{dt} = at\sin t$ —(II)

from (1) and (2)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at\sin t}{at\cos t} = \tan t$$

$$\frac{dy}{dx} = \tan t$$

$$\frac{d^2y}{dx^2} = \sec^2 t \quad \frac{dt}{dx}$$

$$= \frac{1}{\cos^2 t} \cdot \frac{1}{at \cos t}$$

Put $t = \pi/4$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 \pi/4} \cdot \frac{1}{a \cdot \frac{\pi}{4} \cdot \cos\frac{\pi}{4}}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} \cdot \frac{1}{a \cdot \frac{\pi}{4} \cdot \frac{1}{\sqrt{2}}}$$

$$= \frac{1}{a \cdot \frac{\pi}{4} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}}}$$

$$\frac{d^2y}{dx^2} = \frac{8\sqrt{2}}{a\pi} \quad \text{Ans.}$$

(2) $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} \frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{y}} \frac{dy}{dx} = \frac{-1}{\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}} \quad \text{—— (1)}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{(\sqrt{x})\frac{1}{2\sqrt{y}}\cdot\frac{dy}{dx} - \sqrt{y}\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}\right]$$

$$= \frac{-1}{2x}\left[\frac{\sqrt{x}}{\sqrt{y}}\left(\frac{-\sqrt{y}}{\sqrt{x}}\right) - \frac{\sqrt{y}}{\sqrt{x}}\right]$$

$$= \frac{-1}{2x}\left[-1 - \frac{\sqrt{y}}{\sqrt{x}}\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{2x}\left[\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}}\right]$$

$$= \frac{1}{2x\cdot\sqrt{x}}\cdot\sqrt{a}.$$

$$\frac{d^2y}{dx^2} = \frac{1}{2x^{3/2}}\cdot\sqrt{a}.$$

Now put $x=a$

$$\frac{d^2y}{dx^2} = \frac{1}{2(a)^{3/2}} \cdot (a)^{1/2}$$

$$= \frac{1}{2(a)^{3/2-1/2}} = \frac{1}{2a} \text{ Ans.}$$