

CALCULUS : Successive Differentiation

1. $x = a (\cos t + t \sin t)$

$$y = a (\sin t - t \cos t)$$

Compute $\frac{d^2y}{dx^2}$ at $t = \pi/4$

2. $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Find the value of $\frac{d^2y}{dx^2}$ at $x = a$.

$$x = a(\cos t + t \sin t)$$

$$\frac{dx}{dt} = a(-\cancel{\sin t} + t \cos t + \cancel{\sin t})$$

$$\frac{dx}{dt} = at \cos t \quad \text{--- (i)}$$

$$y = a(\sin t - t \cos t)$$

$$\frac{dy}{dt} = a[\cos t - (t(-\sin t) + \cos t)]$$

$$= a(\cos t + t \sin t - \cos t)$$

$$\frac{dy}{dt} = at \sin t \quad \text{--- (ii)}$$

from ① and ②

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at \sin t}{at \cos t} = \tan t$$

$$\frac{dy}{dx} = \tan t$$

$$\frac{d^2y}{dx^2} = \sec^2 t \frac{dt}{dx}$$

$$= \frac{1}{\cos^2 t} \cdot \frac{1}{a t \cos t}$$

Put $t = \pi/4$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 \pi/4} \cdot \frac{1}{a \cdot \frac{\pi}{4} \cdot \cos \frac{\pi}{4}}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} \cdot \frac{1}{a \cdot \frac{\pi}{4} \cdot \frac{1}{\sqrt{2}}}$$



$$\frac{d^2y}{dx^2} =$$

$$= \frac{1}{a \cdot \frac{\pi}{4} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}}}$$

$$\frac{8\sqrt{2}}{a\pi}$$

Ans.

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②

$$\sqrt{x} + \sqrt{y} = \sqrt{a}.$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} \frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{y}} \frac{dy}{dx} = \frac{-1}{\sqrt{x}}$$



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$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}} \quad \text{--- ①}$$

$$\frac{d^2y}{dx^2} = \left[\frac{(\sqrt{x}) \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} - \sqrt{y} \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} \right]$$

$$= \frac{-1}{2x} \left[\frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{y}}} \left(\frac{-\cancel{\sqrt{y}}}{\cancel{\sqrt{x}}} \right) - \frac{\sqrt{y}}{\sqrt{x}} \right]$$

$$= \frac{-1}{2x} \left[-1 - \frac{\sqrt{y}}{\sqrt{x}} \right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{2x} \left[\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x}} \right]$$

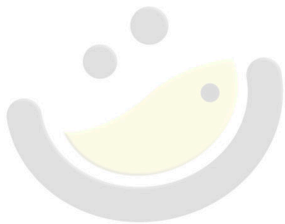
$$= \frac{1}{2x \cdot \sqrt{x}} \cdot \sqrt{a}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2x^{3/2}} \cdot \sqrt{a}$$

Now put x=a

$$\frac{d^2y}{dx^2} = \frac{1}{2(a)^{3/2}} \cdot (a)^{1/2}$$

$$= \frac{1}{2(a)^{3/2-1/2}} = \frac{1}{2a} \text{ Ans.}$$



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