CALCULUS : Successive Differentiation

1. $y = \cosh(\log x) + \sinh(\log x)$

Prove that $y_n = 0$ for $n > 1$.

2. $y = e^{ax} \cosh bx$

Prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 - b^2)y = 0$

1. $y = \cosh(\log x) + \sinh(\log x)$

$y_1 = \sinh(\log x) \cdot \frac{d}{dx}(\log x) + \cosh(\log x) \frac{d}{dx}\log x$

$$= \sinh(\log x) \cdot \frac{1}{x} + \cosh(\log x) \cdot \frac{1}{x}$$

$$y_1 = \frac{1}{x} \left\{ \sinh(\log x) + \cosh(\log x) \right]$$

$$y_1 = \frac{1}{x} y$$

$$\underset{1}{x} \, \underset{2}{y_1} = y$$

Differentiate Both Side.

$$x y_2 + \cancel{y_1} = \cancel{y_1}$$

$$x y_2 = 0$$

$y_2 = 0$ where $\underline{\underline{2 > 1}}$.

Similarly $y_n = 0$ when $\underline{\underline{n > 1}}$.

Hence Proved.

$$y = \underset{1}{e^{ax}} \; \underset{2}{\cosh bx} \quad \text{--- (1)}$$

$$\frac{dy}{dx} = e^{ax} \sinh bx \cdot b + \cosh bx \; e^{ax} \cdot a$$

$$= \underset{1}{e^{ax}} \underset{2}{(b \sinh bx + a \cosh bx)} \quad \text{--- (2)}$$

$$\frac{d^2y}{dx^2} = e^{ax} [b \cosh bx \cdot b + a \sinh bx \cdot b) +$$

$$(b \sinh bx + a \cosh bx) e^{ax} \cdot a$$

$$= e^{ax} [b^2 \cosh bx + ab \sinh bx + ab \sinh bx$$

$$+ a^2 \cosh bx]$$

$$= e^{ax}[b^2 \cosh bx + 2ab \sinh bx + a^2 \cosh bx] \quad \text{--- (11)}$$

L.H.S

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2-b^2)y$$

$$e^{ax}(b^2 \cosh bx + 2ab \sinh bx + a^2 \cosh bx)$$

$$-2a(e^{ax}(b \sinh bx + a \cosh bx)) + (a^2-b^2) e^{ax} \cosh bx$$

(from ①, ② and ③)

$$b^2e^{ax}\cancel{\cosh bx} + 2ab\,e^{ax}\cancel{\sinh bx} + a^2e^{ax}\cosh bx$$

$$-2ab\,e^{ax}\cancel{\sinh bx} - 2a^2e^{ax}\cosh bx$$

$$+a^2e^{ax}\cosh bx - b^2e^{ax}\cancel{\cosh bx}.$$

$= 0 = R.H.S$

Hence $L.H.S = R.H.S$