If $f(x) = \tan x$, show that

$$f^{V}(0) = 16$$

Sol

$$f(x) = \tan x. \quad \text{--- } ①$$

$$f'(x) = \sec^2 x = 1 + \tan^2 x. \quad \text{--- } ①$$

$$f''(x) = 0 + 2\tan x \frac{d}{dx} \tan x$$

$$= 2\tan x \cdot \sec^2 x$$

$$= 2\tan x\,(1 + \tan^2 x)$$

$$= 2\tan x + 2\tan^3 x$$

$$f''(x) = 2f(x) + 2[f(x)]^3 \quad \text{— (iii)}$$

$$f'''(x) = 2f'(x) + 2\times 3(f(x))^{3-1}\frac{d}{dx}f(x)$$

$$= 2f'(x) + 6(\underbrace{f(x)}_{\mathrm{I}})^2\,\underbrace{f'(x)}_{\mathrm{II}} \quad \text{— IV}$$

$$f^{IV}(x) = 2f''(x) + 6\left[(f(x))^2 f''(x) + f'(x)\, 2f(x)\frac{d}{dx}f(x)\right]$$

$$f^{IV}(x) = 2f''(x) + 6\underset{1}{f''(x)}\underset{2}{(f(x))^2} + 12\underset{1}{f(x)}\underset{2}{(f'(x))^2} \quad \text{— V}$$

$$f^{V}(x) = 2f'''(x) + 6\left[f''(x)\,2(f(x))^{2-1}\frac{d}{dx}f(x) + (f(x))^2 f'''(x)\right]$$

$$+ 12\left[f(x)\,2f'(x)\frac{d}{dx}f'(x) + (f'(x))^2 f'(x)\right]$$

$$= 2f'''(x) + 12f''(x)f(x)f'(x) + 6(f(x))^2 f'''(x)$$
(2)

$$+ 24f(x)f'(x)f''(x) + 12(f'(x))^3 \quad \text{— } \underline{\mathrm{VI}}$$

Put $x = 0$ in I to VI.

$f(0) = 0$ $\quad$ $f''(0) = 0$ $\quad$ $f^{IV}(0) = 0$

$f'(0) = 1$ $\quad$ $f'''(0) = 2$ $\quad$ $f^{V}(0) = 16$

$f^{V}(0) = 16$ Hence Proved