

## Trigonometry And Matrices : Applications Of De Moivre's Theorem

show that the roots of the equation

$$(1+x)^{2n} + (1-x)^{2n} = 0 \text{ are given by}$$

$$\pm i \tan \frac{(2r-1)\pi}{4n} \text{ where } r=0, 1, 2, \dots, n$$

Sol.

$$(1+x)^{2n} + (1-x)^{2n} = 0$$

$$(1+x)^{2n} = -(1-x)^{2n}$$

$$\left(\frac{1+x}{1-x}\right)^{2n} = (-1)$$

$$\frac{1+x}{1-x} = (-1)^{1/2n}$$

$$\frac{1+x}{1-x} = (\cos \pi + i \sin \pi)^{1/2n}$$

$$= (\cos (2n\pi + \pi) + i \sin (2n\pi + \pi))^{1/2n}$$

$$= \cos \frac{(2n+1)\pi}{2n} + i \sin \frac{(2n+1)\pi}{2n}$$

$$\frac{1+x}{1-x} = \cos 0 + i \sin 0 \quad 0 = \frac{(2n+1)\pi}{2n}$$

By Componendo and dividendo

$$\frac{1+x + (1-x)}{1+x - (1-x)} = \frac{\cos\theta + i \sin\theta + 1}{\cos\theta + i \sin\theta - 1}$$

$$\frac{2}{2x} = \frac{(1 + \cos\theta) + i \sin\theta}{-1(1 - \cos\theta) + i \sin\theta} \quad \begin{array}{l} 1 - \cos 2\theta \\ = 2 \sin^2\theta \end{array}$$

$$= \frac{2 \cos^2\theta/2 + i 2 \sin\theta/2 \cos\theta/2}{-1(2 \sin^2\theta/2) + i 2 \sin\theta/2 \cos\theta/2} \quad \begin{array}{l} 1 + \cos 2\theta \\ = 2 \cos^2\theta \\ \sin 2\theta \\ 2 \sin\theta \cos\theta \end{array}$$

$$= \frac{2 \cos \theta / 2 [\cos \theta / 2 + i \sin \theta / 2]}{2i \sin \theta / 2 [-\frac{1}{i} \sin \frac{\theta}{2} + \cos \theta / 2]}$$

$$= \frac{1}{i} \cot \theta / 2 [\cos \theta / 2 + i \sin \theta / 2]$$

$$= \frac{1}{i} \cot \theta / 2 [\cancel{\cos \theta / 2 + i \sin \theta / 2}]{\cancel{(\cos \theta / 2 + i \sin \theta / 2)}}$$

$$\frac{1}{x} = \frac{1}{i} \cot \frac{\theta}{2}$$

$$x = i \tan \frac{\theta}{2}$$

$$= i \tan \frac{(2r+1)\pi}{2n} \times \frac{1}{2}$$

$$= i \tan \frac{(2r+1)\pi}{4n}$$

$$r = 0, 1, 2, \dots, (n-1)$$

$$x = \pm i \tan \frac{(2r-1)\pi}{4n} \quad r = (0, 1, 2, \dots, n)$$

Hence proved.