

Trigonometry And Matrices : Applications Of De Moivre's Theorem

Prove that the roots of the equation $(x-1)^n = x^n$ are

$$\frac{1}{2} \left[1 + i \cot \frac{r\pi}{2} \right] \text{ where } r = 0, 1, 2, \dots, (n-1)$$

Sol

$$(x-1)^n = x^n$$

$$\left(\frac{x-1}{x} \right)^n = 1$$

$$\frac{x-1}{x} = (1)^{1/n}$$

$$= (\cos 0 + i \sin 0)^{1/n}$$

$$\frac{x-1}{x} = [\cos(2r\pi + 0) + i \sin(2r\pi + 0)]^{1/n}$$

where r

$= 0, 1, 2, \dots, (n-1)$

$$\frac{x}{x} - \frac{1}{x} = \cos \frac{2r\pi}{n} + i \sin \frac{2r\pi}{n}$$

$$1 - \frac{1}{x} = \cos \theta + i \sin \theta \quad \left[\theta = \frac{2r\pi}{n} \right]$$

$$\frac{-1}{x} = -1 + \cos\theta + i \sin\theta$$

$$\frac{1}{x} = 1 - \cos\theta - i \sin\theta$$

$$x = \frac{1}{1 - \cos\theta - i \sin\theta}$$

$$= \frac{1}{2 \sin^2 \theta/2 - i [2 \sin\theta/2 \cos\theta/2]}$$

$$1 - \cos 2\theta$$

$$= 2 \sin^2 \theta$$

$$\sin 2\theta$$

$$= 2 \sin\theta \cos\theta$$

$$= \frac{1}{2i \sin \theta/2 \left[\frac{1}{i} \sin \theta/2 - \cos \theta/2 \right]}$$

$$= \frac{1}{2i \sin \theta/2 \left[-i \sin \theta/2 - \cos \theta/2 \right]}$$

$$= \frac{1}{-2i \sin \theta/2 \left(\cos \theta/2 + i \sin \theta/2 \right)}$$

$$= \frac{i}{-2i^2 \sin \theta/2} (\cos \theta/2 + i \sin \theta/2)^{-1}$$

$$= \frac{i}{2 \sin \theta/2} (\cos \theta/2 - i \sin \theta/2)$$

$$= \frac{1}{2} \left[i \cot \frac{\theta}{2} + 1 \right]$$

$$= \frac{1}{2} \left[i \cot \frac{2\pi n}{n} x \frac{1}{2} + 1 \right]$$

$$\cos(-\theta)$$

$$= \cos \theta$$

$$\sin(-\theta)$$

$$= -\sin \theta.$$

$$= \frac{1}{2} \left[i \cot \frac{r\pi}{n} + 1 \right] \text{ where } r = 0, 1, 2, \dots, (n-1)$$

Hence Proved



OMG MATHS!
The poetry of logical ideas.