Trigonometry And Matrices : Applications Of De Moivre's Theorem

Solve $(1+z)^6 + z^6 = 0$ where z is a Complex No.

$$(1+z)^6 = -z^6$$

$$\left(\frac{1+z}{z}\right)^6 = -1$$

$$\left(\frac{1+z}{z}\right) = (-1)^{1/6}$$

$$\frac{1+z}{z} = (\cos\pi + i\sin\pi)^{1/6}$$

$$\frac{1}{z} + \frac{z}{z} = \left(\cos(2n\pi+\pi) + i\sin(2n\pi+\pi)\right)^{1/6}$$

$$n = 0, 1, 2, 3, 4, 5$$

$$1 + \frac{1}{z} = \cos\frac{(2n+1)\pi}{6} + i\sin\frac{(2n+1)\pi}{6}$$

$$\frac{1}{z} = -1 + \cos\theta + i\sin\theta \quad \left\{ \text{where } \theta = \frac{(2n+1)\pi}{6} \right]$$

$$z = \frac{1}{-1 + \cos\theta + i\sin\theta}$$

$$z = \frac{1}{-1(1-\cos\theta) + i\sin\theta}$$

$$= \frac{1}{-1\left(2\sin^2\theta/2\right) + 2i\sin\theta/2\,\cos\frac{\theta}{2}}$$

$$= \frac{1}{2i\sin\theta/2\left[-\frac{1}{i}\sin\theta/2 + \cos\theta/2\right]}$$

$1-\cos 2\theta$

$= 2\sin^2\theta$

$\sin 2\theta$

$= 2\sin\theta\cos\theta$

$$= \frac{1}{2i\sin\frac{\theta}{2}\left[\cos\theta/2 + i\sin\theta/2\right]}$$

$$= \frac{1}{2i\sin\theta/2}\left[\cos\theta/2 + i\sin\theta/2\right]^{-1}$$

$\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta.$

$$= \frac{i}{2i^2\sin\theta/2}\left[\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right]$$

$$= \frac{-i}{2\sin\theta/2}\left\{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right\}$$

$$= \frac{-1}{2}\left[i\cot\frac{\theta}{2} + 1\right]$$

$$= \frac{-1}{2}\left[1 + i\cot\frac{(2n+1)\pi}{6}\right] \quad n = 0, 1, 2, 3, 4, 5$$

Ans.