Trigonometry And Matrices : Applications Of De Moivre's Theorem

Solve $x^{10} + x^8 + x^6 + x^4 + x^2 + 1 = 0$

Multiply with $(x^2 - 1)$

$$(x^2 - 1)(x^{10} + x^8 + x^6 + x^4 + x^2 + 1) = 0$$

$$x^{12} + \cancel{x^{10}} + \cancel{x^8} + \cancel{x^6} + \cancel{x^4} + \cancel{x^2} - \cancel{x^{10}} - \cancel{x^8} - \cancel{x^6} - \cancel{x^4} - \cancel{x^2} - 1 = 0$$

$$x^{12} - 1 = 0$$

$$x = (1)^{1/12}$$

$$x = (\cos 0 + i\sin 0)^{1/12}$$

$$= \left[\cos(2n\pi+0) + i\sin(2n\pi+0)\right]^{\frac{1}{12}}$$

$n = 0, 1, 2, ----, 11$

$$x = \cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}$$ where

$n = 0, 1, 2, 3, ----, 11$

$\cos\pi/3 + i\sin\pi/3$

$$x = (\cos 0 + i\sin 0), \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right), \left(\cos\frac{\cancel{2}\pi}{\cancel{6}\,3} + i\sin\frac{\cancel{2}\pi}{\cancel{6}\,3}\right),$$

$$\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right), \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right),$$

$$\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right), \left(\cos\pi + i\sin\pi\right),$$

$$\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right), \left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right),$$

$$\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right), \left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right),$$

$$\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}$$

$$x = 1, \quad \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}, \quad \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}, \quad i,$$

$$\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3},\ \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6},\ -1\Big\},$$

$$\Big(\cos\left(2\pi - \frac{5\pi}{6}\right) + i\sin\left(2\pi - \frac{5\pi}{6}\right)\Big),$$

$$\cos\left(2\pi - \frac{2\pi}{3}\right) + i\sin\left(2\pi - \frac{2\pi}{3}\right)$$

$$\cos\left(2\pi - \pi/2\right) + i\sin\left(2\pi - \pi/2\right)$$

$$\cos\left(2\pi - \pi/3\right) + i\sin\left(2\pi - \pi/3\right)$$

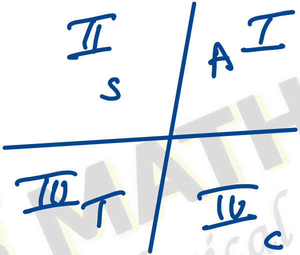
$$\cos\left(2\pi - \pi/6\right) + i\sin\left(2\pi - \pi/6\right)$$

$x = 1,\ \cos\frac{\pi}{6} \pm i\sin\frac{\pi}{6},$

$\cos\frac{\pi}{3} \pm i\sin\frac{\pi}{3},\ \pm i,$

$\cos\frac{2\pi}{3} \pm i\sin\frac{2\pi}{3},$

$\cos\frac{5\pi}{6} \pm i\sin 5\pi/6,\ -1$



Cos +ve
Sin −ve

Roots of Given equation are $\cos\frac{\pi}{6} \pm i\sin\frac{\pi}{6}$, $\left(\cos\frac{\pi}{3} \pm i\sin\pi/3\right)$, $\left(\cos 2\pi/3 \pm i\sin 2\pi/3\right)$, $\pm i$,

$$\left(\cos\frac{5\pi}{6} \pm i\sin\frac{5\pi}{6}\right)$$ Ans.