Pascal's Rule

$$x = \cos\theta + i\sin\theta$$

$$\frac{1}{x} = \frac{1}{\cos\theta + i\sin\theta} =$$

$$= \cos\theta - i\sin\theta$$

$$x + \frac{1}{x} = \cos\theta + i\cancel{\sin\theta} + \cos\theta - i\cancel{\sin\theta}$$

$$= 2\cos\theta$$

$$x - \frac{1}{x} = \cancel{\cos\theta} + i\sin\theta - \cancel{\cos\theta} + i\sin\theta$$

$$= 2i\sin\theta$$

$$x^m = (\cos\theta + i\sin\theta)^m$$

$$= \cos m\theta + i\sin m\theta$$

$$\frac{1}{x^m} = \cos m\theta - i\sin m\theta$$

$$x^m + \frac{1}{x^m} = 2\cos m\theta$$

$$x^m - \frac{1}{x^m} = 2i\sin m\theta.$$

$$(x+a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + - - - - + {}^nC_n\, x^{n-n} a^n.$$

$$\left(x+\frac{1}{x}\right)^6 = x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 26 28 8 1

$$\cos^7\theta = \frac{1}{2^6}\left(\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta\right)$$

Sol. $x = \cos\theta + i\sin\theta$

$x^m = \cos m\theta + i\sin m\theta$

$\frac{1}{x} = \cos\theta - i\sin\theta$

$\frac{1}{x^m} = \cos m\theta - i\sin m\theta$

$x + \frac{1}{x} = 2\cos\theta$ $\qquad x^m + \frac{1}{x^m} = 2\cos m\theta.$

$$\left(x+\frac{1}{x}\right)^7 = 2^7 \cos^7\theta$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 26 28 8 1

$$\left(x+\frac{1}{x}\right)^7 = x^7 + 7x^5 + 21x^3 + 35x + \frac{35}{x} + \frac{21}{x^3} + \frac{7}{x^5} + \frac{1}{x^7}$$

$$2^7\cos^7\theta = \left(x^7+\frac{1}{x^7}\right) + 7\left(x^5+\frac{1}{x^5}\right) + 21\left(x^3+\frac{1}{x^3}\right)$$

$$+ 35\left(x + \frac{1}{x}\right)$$

$$= 2\cos 7\theta + 7\times 2\cos 5\theta + 21\times 2\cos 3\theta + 35\times 2\cos\theta$$

$$2^7\cos^7\theta = 2\left(\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta\right)$$

$$\cos^7\theta = \frac{2}{2^7}\{(1)\} = \frac{1}{2^6}\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta.$$

Hence L.H.S = R.H.S

Proved.

$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta.$

Sol

$x = \cos\theta + i\sin\theta$

$x^m = \cos m\theta + i\sin m\theta$

$\frac{1}{x} = \cos\theta - i\sin\theta$

$\frac{1}{x^m} = \cos m\theta - i\sin m\theta$

$$x - \frac{1}{x} = 2i\sin\theta.$$

$$x^n - \frac{1}{x^n} = 2i\sin n\theta.$$

$$i^5 = i$$

$$\left(x - \frac{1}{x}\right)^5 = 2^5 i^5 \sin^5\theta.$$

$$= 2^5 i \sin^5\theta$$

$$\left(x - \frac{1}{x}\right)^5 = ?$$

$$\left(x-\frac{1}{x}\right)^5 = x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 26 28 8 1

$$2^5 i \sin^5\theta = \left(x^5 - \frac{1}{x^5}\right) - 5\left(x^3 - \frac{1}{x^3}\right) + 10\left(x - \frac{1}{x}\right)$$

$$= 2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)$$

$$= 2i(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$$

$$\sin^5\theta = \frac{2\cancel{i}}{2^5\cancel{i}}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right)$$

$$= \frac{1}{2^4}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right)$$

$$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$$