Pascal's Triangle

Prove that

$$\cos^6\theta \sin^4\theta = 2^{-9}[\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6]$$

Sol

$$x = \cos\theta + i\sin\theta$$

$$\frac{1}{x} = (\cos\theta + i\sin\theta)^{-1}$$

$$= \cos\theta - i\sin\theta$$

$$x^m = (\cos\theta + i\sin\theta)^m$$

$$= \cos m\theta + i\sin m\theta$$

$$\frac{1}{x^m} = \cos m\theta - i\sin m\theta$$

$$x + \frac{1}{x} = \cos\theta + i\cancel{\sin\theta} + \cos\theta - i\cancel{\sin\theta}$$

$$= 2\cos\theta$$

$$x^m + \frac{1}{x^m} = 2\cos m\theta.$$

$$x - \frac{1}{x} = \cancel{\cos\theta} + i\sin\theta - \cancel{\cos\theta} + i\sin\theta$$

$$= 2i\sin\theta$$

$\cos^6\theta \sin^4\theta$

$$\left(x+\frac{1}{x}\right)^6 = 2^6\cos^6\theta$$

$$\left(x-\frac{1}{x}\right)^4 = (2i\sin\theta)^4 = 2^4 i^4 \sin^4\theta$$

$$= 2^4\sin^4\theta$$

$$\left(x+\frac{1}{x}\right)^6\left(x-\frac{1}{x}\right)^4 = 2^6 \cdot 2^4 \cos^6\theta \sin^4\theta$$

$$= 2^{10}\cos^6\theta\sin^4\theta$$

$$\left(x+\frac{1}{x}\right)^6\left(x-\frac{1}{x}\right)^4$$

1 6 15 20 15 6 1

① 1 −1

1 6 15 20 15 6 1

−1 −6 −15 −20 −15 −6 −1

1 5 9 5 −5 −9 −5 −1

② 1 −1

1 5 9 5 −5 −9 −5 −1

−1 −5 −9 −5 5 9 5 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

	1	4	4	-4	-10	-4	4	4	1	

3	1	-1									
	1	4	4	-4	-10	-4	4	4	1		
		-1	-4	-4	+4	+10	+4	-4	-4	-1	
	1	3	0	-8	-6	6	8	0	-3	-1	

4	1	-1									
	1	3	0	-8	-6	6	8	0	-3	-1	
		-1	-3	0	+8	+6	-6	-8	~~0~~	+3	+1
	1	2	-3	-8	2	12	2	-8	-3	2	1

$$x^{10} + 2x^8 - 3x^6 - 8x^4 + 2x^2 + 12 + \frac{2}{x^2} - \frac{8}{x^4} - \frac{3}{x^6}$$

$$+ \frac{2}{x^8} + \frac{1}{x^{10}}$$

$$\left(x^{10} + \frac{1}{x^{10}}\right) + 2\left(x^8 + \frac{1}{x^8}\right) + (-3)\left(x^6 + \frac{1}{x^6}\right)$$

$$-8\left(x^4 + \frac{1}{x^4}\right) + 2\left(x^2 + \frac{1}{x^2}\right)$$

$$= 2\cos 10\theta + 2.(2\cos 8\theta) - 3(2\cos 6\theta) - 8(2\cos 4\theta) + 12$$

$$+ 2(2\cos 2\theta) + 12$$

$$= 2\,[\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6]$$

$$2^{10}\cos^6\theta\,\sin^4\theta = 2\,[\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6)$$

$$\cos^6\theta\,\sin^4\theta = \frac{1}{2^9}(\text{4 same})$$

Hence Proved.