

## Trigonometry And Matrices : Applications Of De Moivre's Theorem

Prove that

Pascal's Triangle

$$\cos^6 \theta \sin^4 \theta = 2^{-9} [\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6]$$

So

$$x = \cos \theta + i \sin \theta$$

$$\frac{1}{x} = (\cos \theta + i \sin \theta)^{-1}$$

$$= \cos \theta - i \sin \theta$$

$$x^m = (\cos \theta + i \sin \theta)^m$$

$$= \cos m\theta + i \sin m\theta$$

$$\frac{1}{x^m} = \cos m\theta - i \sin m\theta$$

$$x + \frac{1}{x^m} = \cos \theta + i \sin \theta + \cos \theta - i \sin \theta \\ = 2 \cos \theta$$

$$x^m + \frac{1}{x^m} = 2 \cos m\theta.$$

$$x - \frac{1}{x^m} = \cos \theta + i \sin \theta \\ - \cos \theta + i \sin \theta \\ = 2i \sin \theta$$

$$\cos^6 \theta \sin^4 \theta$$

$$\left(x + \frac{1}{x}\right)^6 = 2^6 \cos^6 \theta$$

$$\left(x - \frac{1}{x}\right)^4 = (2 i \sin \theta)^4 = 2^4 i^4 \sin^4 \theta = 2^4 \sin^4 \theta$$

$$\left(x + \frac{1}{x}\right)^6 \left(x - \frac{1}{x}\right)^4 = 2^6 \cdot 2^4 \cos^6 \theta \sin^4 \theta \\ = 2^{10} \cos^6 \theta \sin^4 \theta$$

$$\left(x + \frac{1}{x}\right)^6 \left(x - \frac{1}{x}\right)^4$$

1      6      15      20      15      6      1

① 1      -1

$$\begin{array}{r}
 1 & 6 & 15 & 20 & 15 & 6 & 1 \\
 -1 & -6 & -15 & -20 & -15 & -6 & -1 \\
 \hline
 1 & 5 & 9 & 5 & -5 & -9 & -5 & -1
 \end{array}$$

② 1      -1

$$\begin{array}{r}
 1 & -5 & 9 & -5 & -5 & -9 & -5 & -1 \\
 -1 & -5 & -5 & -9 & -5 & 5 & 9 & 5 & 1
 \end{array}$$

1										
1	1									
1	2	1								
1	3	3	1							
1	4	6	4	1						
1	5	10	10	5	1					
1	6	15	20	15	6	1				
1	7	21	35	35	21	7	1			
1	8	28	56	70	56	28	8	1		
1	9	36	84	126	126	84	36	9	1	
1	10	45	120	210	252	210	120	45	10	1

$$\begin{array}{ccccccccc} & 1 & 4 & & 4 & -4 & -10 & -4 & 4 & 4 & 1 \\ \hline \end{array}$$

③  $\begin{array}{r} 1 \quad -1 \\ \hline \end{array}$

$$\begin{array}{ccccccccc} & 1 & 4 & 4 & -4 & -10 & -4 & 4 & 4 & 1 \\ & -1 & -4 & -4 & +4 & +10 & +4 & -4 & -4 & -1 \\ \hline & 1 & 3 & 0 & -8 & -6 & 6 & 8 & 0 & -3 & -1 \end{array}$$

④  $\begin{array}{r} 1 \quad -1 \\ \hline \end{array}$

$$\begin{array}{ccccccccc} & 1 & 3 & 0 & -8 & -6 & 6 & 8 & 0 & -3 & -1 \\ & -1 & -3 & 0 & +8 & +6 & -6 & -8 & 0 & +3 & +1 \\ \hline & 1 & 2 & -3 & -8 & 2 & 12 & 2 & -8 & -3 & 2 & 1 \end{array}$$

$$x^{10} + 2x^8 - 3x^6 - 8x^4 + 2x^2 + 12 + \frac{2}{x^2} - \frac{8}{x^4} - \frac{3}{x^6}$$

$$\left( x^{10} + \frac{1}{x^{10}} \right) + 2 \left( x^8 + \frac{1}{x^8} \right) + (-3) \left( x^6 + \frac{1}{x^6} \right) \\ + \frac{2}{x^2} + \frac{1}{x^{10}}$$

$$- 8 \left( x^4 + \frac{1}{x^4} \right) + 2 \left( x^2 + \frac{1}{x^2} \right)$$

$$= 2 \cos 100^\circ + 2 \cdot [2 \cos 80^\circ] - 3 \left( 2 \cos \frac{12}{60}^\circ \right) - 8 \left( 2 \cos 40^\circ \right) \\ + 2 \left( 2 \cos 20^\circ \right) + 12$$

$$= 2 (\cos 100 + 2 \cos 80 - 3 \cos 60 - 8 \cos 40 \\ + 2 \cos 20 + 6)$$

$$2^{10} \cos^6 \theta \sin^4 \theta = 2 (\cos 100 + 2 \cos 80 - 3 \cos 60 - \\ 8 \cos 40 + 2 \cos 20 + 6)$$

$$\cos^6 \theta \sin^4 \theta = \frac{1}{2^9} \quad (4 \text{ terms})$$

Hence Proved.