Express $\cos^8\theta$ in the series of cosines of multiples of θ.

Sol

$$x = \cos\theta + i\sin\theta$$

$$\frac{1}{x} = \cos\theta - i\sin\theta$$

$$x^m = (\cos\theta + i\sin\theta)^m$$

$$= \cos m\theta + i\sin m\theta$$

$$\frac{1}{x^m} = \cos m\theta - i\sin m\theta$$

$$\cos^8\theta = ?$$

$$\left(x + \frac{1}{x}\right) = \cos\theta + i\cancel{\sin\theta} + \cos\theta - i\cancel{\sin\theta}$$

$$= 2\cos\theta$$

$$x^m + \frac{1}{x^m} = 2\cos m\theta$$

$$(2\cos\theta)^8 = \left(x + \frac{1}{x}\right)^8$$

$$2^8\cos^8\theta = \left(x + \frac{1}{x}\right)^8$$

$$\left(x+\frac{1}{x}\right)^8 = x^8 + 8x^6 + 28x^4 + 56x^2 + 70 + \frac{56}{x^2} + \frac{28}{x^4} + \frac{8}{x^6} + \frac{1}{x^8}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

$\boxed{1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1}$

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

$$\left(x+\frac{1}{x}\right)^8 = \left(x^8 + \frac{1}{x^8}\right) + 8\left(x^6 + \frac{1}{x^6}\right) + 28\left(x^4 + \frac{1}{x^4}\right) + 56\left(x^2 + \frac{1}{x^2}\right) + 70$$

$$2^8\cos^8\theta = 2\cos 8\theta + 8(2\cos 6\theta) + 28(2\cos 4\theta) + 56(2\cos 2\theta) + 70$$

$$2^8\cos^8\theta = 2(\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35)$$

$$\cos^8\theta = \frac{1}{2^7}(\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35)$$ Ans