

Trigonometry And Matrices : Successive Differentiation

Find the fourth derivative of
 $y = \log \sqrt{3x+4}$

$$y = \log (3x+4)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log (3x+4)$$

$$\frac{dy}{dx} = \frac{1}{2} \left[\frac{1}{3x+4} \frac{d}{dx} (3x+4) \right]$$

$$\log x^n = n \log x.$$

$$\log x = \frac{1}{x}$$

$$= \frac{1}{2} \cdot \frac{1}{(3x+4)} \quad (3)$$

$$\frac{dy}{dx} = \frac{3}{2} \cdot (3x+4)^{-1}$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{3}{2} \left[(-1) (3x+4)^{-1-1} \right] \frac{d}{dx} (3x+4) \\ &= \frac{-3}{2} (3x+4)^{-2} \quad (3)\end{aligned}$$

$$= -\frac{9}{2} (3x+4)^{-2}$$

$$\frac{d^3y}{dx^3} = -\frac{9}{2} \left[(-2) (3x+4)^{-2-1} \frac{d}{dx} (3x+4) \right]$$

$$= -\frac{9}{2} \left[-2 (3x+4)^{-3} (3) \right]$$

$$= 27 (3x+4)^{-3}$$

$$\frac{d^4y}{dx^4} = 27 \left[-3 (3x+4)^{-3-1} \frac{d}{dx} (3x+4) \right]$$

$$= 27 \left[-3 (3x+4)^{-4} (3) \right]$$

$$= -243 (3x+4)^{-4} = \frac{-243}{(3x+4)^4}$$

$$\frac{d^4y}{dx^4} = \frac{-243}{(3x+4)^4} \quad \text{ans.}$$