Solve the equation

$$x^4 + x^3 + x^2 + x + 1 = 0$$

Sol.

Multiply with $(x-1)$

$$(x-1)(x^4 + x^3 + x^2 + x + 1) = 0$$

$$x^5 + \cancel{x^4} + \cancel{x^3} + \cancel{x^2} + \cancel{x} - \cancel{x^4} - \cancel{x^3} - \cancel{x^2} - \cancel{x} - 1 = 0$$

$$x^5 - 1 = 0$$

$$x^5 = 1$$

$$x = (1)^{1/5}$$

$$x = (\cos 0 + i \sin 0)^{1/5}$$

$$x = \{\cos(2n\pi + 0) + i \sin(2n\pi + 0)\}^{1/5}$$

$n = 0, 1, 2, 3, 4.$

$$x = \cos\frac{2n\pi}{5} + i \sin\frac{2n\pi}{5}$$

$$= (\cos 0 + i \sin 0),\ \cos\frac{2\pi}{5} + i \sin\frac{2\pi}{5},\ \cos\frac{4\pi}{5} + i \sin\frac{4\pi}{5},$$

$$\cos\frac{6\pi}{5} + i \sin\frac{6\pi}{5},\ \cos\frac{8\pi}{5} + i \sin\frac{8\pi}{5}$$

$$(1),\ \left(\cos\frac{2\pi}{5} + i \sin\frac{2\pi}{5}\right),\ \left(\cos\frac{4\pi}{5} + i \sin\frac{4\pi}{5}\right),$$

$$\left(\cos\frac{4\pi}{5} - i\sin\frac{4\pi}{5}\right), \left(\cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right)$$

$$1, \left(\cos\frac{2\pi}{5} \pm i\sin\frac{2\pi}{5}\right), \left(\cos\frac{4\pi}{5} \pm i\sin\frac{4\pi}{5}\right)$$

Now Roots of given equation are

$$\left(\cos\frac{2\pi}{5} \pm i\sin\frac{2\pi}{5}\right), \left(\cos\frac{4\pi}{5} \pm i\sin\frac{4\pi}{5}\right)$$

Ans