Solve the equation

$$x^4 - x^3 + x^2 - x + 1 = 0$$

Sol. Multiply with $(x+1)$

$$(x+1)(x^4 - x^3 + x^2 - x + 1) = 0$$

$$x^5 - \cancel{x^4} + \cancel{x^3} - \cancel{x^2} + \cancel{x} + \cancel{x^4} - \cancel{x^3} + \cancel{x^2} - \cancel{x} + 1 = 0$$

$$x^5 + 1 = 0$$

$$x^5 = -1$$

$$x = (-1)^{1/5}$$

$$x = (\cos\pi + i\sin\pi)^{1/5}$$

$$= \left(\cos(2n\pi+\pi) + i\sin(2n\pi+\pi)\right)^{1/5}$$

$n = 0, 1, 2, 3, 4$

$$= \cos\left(\frac{2n\pi+\pi}{5}\right) + i\sin\left(\frac{2n\pi+\pi}{5}\right)$$

$$= \cos\left(\frac{(2n+1)\pi}{5}\right) + i\sin\left(\frac{(2n+1)\pi}{5}\right)$$

$n = 0, 1, 2, 3, 4$

$$= \left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right), \left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right), \left(\cos\frac{5\pi}{5} + i\sin\frac{5\pi}{5}\right)$$

$$\left(\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}\right), \left(\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}\right)$$

$$= \left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right), \cos\left(\frac{3\pi}{5}\right) + i\sin\left(\frac{3\pi}{5}\right), -1,$$

$$\left(\cos\frac{3\pi}{5} - i\sin\frac{3\pi}{5}\right), \left(\cos\frac{\pi}{5} - i\sin\frac{\pi}{5}\right)$$

$$\left(\cos\frac{\pi}{5} \pm i\sin\frac{\pi}{5}\right), \left(\cos\frac{3\pi}{5} \pm i\sin\frac{3\pi}{5}\right)$$

$\left(\cos\frac{r\pi}{5} \pm i\sin\frac{r\pi}{5}\right)$ where $r = 1, 3$

Ans.