

## Trigonometry And Matrices : Applications Of De Moivre's Theorem

Solve the equation

$$x^4 - x^3 + x^2 - x + 1 = 0$$

Sol. Multiply with  $(x+1)$

$$(x+1)(x^4 - x^3 + x^2 - x + 1) = 0$$

$$x^5 - \cancel{x^4} + \cancel{x^3} - \cancel{x^2} + \cancel{x} + \cancel{x^4} - \cancel{x^3} + \cancel{x^2} - \cancel{x} + 1 = 0$$

$$x^5 + 1 = 0$$

$$x^5 = -1$$

$$x = (-1)^{1/5}$$

$$x = (\cos \pi + i \sin \pi)^{1/5}$$

$$= (\cos (2n\pi + \pi) + i \sin (2n\pi + \pi))^{1/5}$$

$$n = 0, 1, 2, 3, 4$$

$$= \cos \left( \frac{2n\pi + \pi}{5} \right) + i \sin \left( \frac{2n\pi + \pi}{5} \right)$$

$$= \cos \left( \frac{(2n+1)\pi}{5} \right) + i \sin \left( \frac{(2n+1)\pi}{5} \right)$$

$$n = 0, 1, 2, 3, 4$$

$$= \left( \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right), \left( \cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5} \right), \left( \cos \frac{5\pi}{5} + i \sin \frac{5\pi}{5} \right)$$

$$\left( \cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5} \right), \left( \cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5} \right)$$

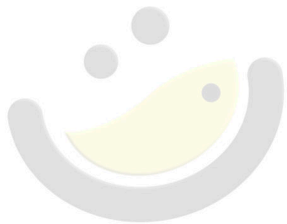
$$= \left( \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right), \cos \left( \frac{3\pi}{5} \right) + i \sin \left( \frac{3\pi}{5} \right), -1,$$

$$\left( \cos \frac{3\pi}{5} - i \sin \frac{3\pi}{5} \right), \left( \cos \frac{\pi}{5} - i \sin \frac{\pi}{5} \right)$$

$$\left( \cos \frac{\pi}{5} \pm i \sin \frac{\pi}{5} \right), \left( \cos \frac{3\pi}{5} \pm i \sin \frac{3\pi}{5} \right)$$

$$\left( \cos \frac{2\pi}{5} \pm i \sin \frac{2\pi}{5} \right) \text{ where } n=1,3$$

Ans.



OMG { MATHS }

The poetry of logical ideas.