

Trigonometry And Matrices : Applications Of De Moivre's Theorem

Primitive Roots of Unity

Show that each primitive 6th root of unity satisfy $x^2 - x + 1 = 0$

Sol.

$$r_4 = \text{cis } \frac{2\pi}{6}$$

$$r_4 = \cos \frac{2\pi}{6} + i \sin \frac{2\pi}{6}$$

$$= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$$

$$r_4^6 = \left(\cos \left(\frac{2\pi}{6} \right) + i \sin \frac{2\pi}{6} \right)^6$$

$$= \cos 2\pi + i \sin 2\pi = \underline{1}$$

$$\underline{\underline{\epsilon_4^6 = 1}}.$$

\therefore for a non integer $k < 6$, ϵ_4^k is primitive root of unity iff $(k, 6) = 1$.

$$\therefore k = 1, 5$$

$\therefore \epsilon_4^1, \epsilon_4^5$ are primitive 6th roots of unity.

$$\epsilon_4^5 = \epsilon_4^6 \cdot \epsilon_4^{-1} = \epsilon_4^{-1} = (\cos \pi/3 + i \sin \pi/3)^{-1}$$

$$\left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}\right)$$

$$\cos(-\theta) = \cos\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$r_4^1 + r_4^5 = \cancel{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} + \cancel{\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}}$$

$$= 2 \cos \frac{\pi}{3} = 2 \times \frac{1}{2} = 1.$$

equation with root r_4^1, r_4^5

$$(x - r_4^1)(x - r_4^5) = 0$$

$$x^2 - r_4^5 x - r_4^1 x + r_4^1 = 0$$

$$x^2 - x(r_1^5 + r_1) + 1 = 0$$

$$x^2 - x + 1 = 0$$

Hence Proved.



OMG{MATHS}
The poetry of logical ideas.