Primitive Roots of Unity

Show that each primitive 6^{th} root of unity satisfy $x^2 - x + 1 = 0$

Sol.

$$r_1 = \text{cis}\,\frac{2\pi}{6}$$

$$r_1 = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}$$

$$= \cos\pi/3 + i\sin\pi/3$$

$$r_1^6 = \left(\cos\left(\frac{2\pi}{6}\right) + i\sin 2\pi/6\right)^6$$

$= \cos 2\pi + i \sin 2\pi = \underline{1}$

$\underline{\underline{r_4^6 = 1.}}$

$\therefore$ for a +ve Integer $K < 6$, r_4^k is primitive roots of unity iff $(K, 6) = 1.$

$\therefore K = 1, 5$

$\therefore r_4^1, r_4^5$ are primitive 6^{th} roots of unity.

$r_4^5 = r_4^6 \cdot r_4^{-1} = r_4^{-1} = \left(\cos \pi/3 + i \sin \pi/3\right)^{-1}$

$$\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$$r_4^1 + r_4^5 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} + \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$$

$$= 2\cos\frac{\pi}{3} = 2 \times \frac{1}{2} = 1.$$

Eluation with root r_4^1, r_4^5

$$(x - r_4)(x - r_4^5) = 0$$

$$x^2 - r_4^5 x - r_4 x + r_4^6 = 0$$

$$x^2 - x(y^5 + y) + 1 = 0$$

$$x^2 - x + 1 = 0$$

Hence Proved