

Trigonometry And Matrices : De Moivre's Theorem

$$1. \left(\frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta} \right)^4 = \cos 8\theta + i \sin 8\theta$$

$$2. \frac{z^{2n} - 1}{z^{2n} + 1} = i \tan n\theta \quad (z = \cos \theta + i \sin \theta)$$

$$3. \left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta} \right)^n = \cos \left(\frac{n\pi}{2} - n\theta \right) + i \sin \left(\frac{n\pi}{2} - n\theta \right)$$

Sol. ①

$$\left(\frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta} \right)^4 =$$

$$\cos 8\theta + i \sin 8\theta$$

$$(\cos \theta + i \sin \theta)^n =$$

$$\cos n\theta + i \sin n\theta.$$

L.H.S

$$\left(\frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta} \right)^4$$

$$\frac{(\cos \theta + i \sin \theta)^4}{(\sin \theta + i \cos \theta)^4} =$$

$$\frac{\cos 4\theta + i \sin 4\theta}{(\cos(\pi/2 - \theta) + i \sin(\pi/2 - \theta))^4}$$

$$= \frac{\cos 40 + i \sin 40}{\cos(2\pi - 40) + i \sin(2\pi - 40)}$$

$$= \frac{\cos 40 + i \sin 40}{\cos 40 - i \sin 40}$$

$$= (\cos 40 + i \sin 40)(\cos 40 - i \sin 40)^{-1}$$

$$= (\cos 40 + i \sin 40)(\cos 40 + i \sin 40)$$

$$\left[\begin{array}{l} \because \cos(-\theta) \\ = \cos \theta \\ \sin(-\theta) \\ = -\sin \theta \end{array} \right.$$

$$= (\cos 40 + i \sin 40)^2$$

$$= \cos 80 + i \sin 80 = \text{R.H.S}$$

Hence proved.

②

$$Z = \cos \theta + i \sin \theta.$$

L.H.S
=

$$\frac{Z^{2n} - 1}{Z^{2n} + 1} = \frac{(\cos \theta + i \sin \theta)^{2n} - 1}{(\cos \theta + i \sin \theta)^{2n} + 1}$$

$$= \frac{\cos 2n\theta + i \sin 2n\theta - 1}{\cos 2n\theta + i \sin 2n\theta + 1}$$

$$= \frac{-1 (1 - \cos 2n\theta) + i \sin 2n\theta}{1 + \cos 2n\theta + i \sin 2n\theta}$$

$$= \frac{-2 \sin^2 n\theta + i \cdot 2 \sin n\theta \cos n\theta}{2 \cos^2 n\theta + i (2 \sin n\theta \cos n\theta)}$$

$$= \frac{2 \sin n\theta \cdot i \left(\frac{-\sin n\theta}{i} + \cos n\theta \right)}{2 \cos n\theta \left[\cos n\theta + i \sin n\theta \right]}$$

$$= \frac{i \sin n\theta \left(\frac{i^2 \sin n\theta}{i} + \cos n\theta \right)}{i \cos n\theta \left(\cos n\theta + i \sin n\theta \right)}$$

$$= \frac{i \tan n\theta \left(\cancel{\cos n\theta} + i \sin n\theta \right)}{\left(\cancel{\cos n\theta} + i \sin n\theta \right)} = i \tan n\theta = \underline{\underline{R.H.S}}$$

3. L.H.S $\left(\frac{1 + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta} \right)^n$

$$= \left(\frac{\sin^2\theta + \cos^2\theta + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta} \right)^n$$

$$= \left(\frac{\sin^2\theta - i^2 \cos^2\theta + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta} \right)^n$$

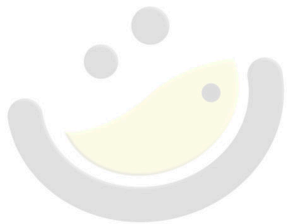
$$= \left(\frac{(\sin \theta + i \cos \theta)(\sin \theta - i \cos \theta) + (\sin \theta + i \cos \theta)^n}{1 + \sin \theta - i \cos \theta} \right)^n$$

$$= \left(\frac{(\sin \theta + i \cos \theta)(\sin \theta - i \cos \theta + 1)}{1 + \sin \theta - i \cos \theta} \right)^n$$

$$= (\sin \theta + i \cos \theta)^n$$

$$= \left(\cos\left(\frac{\pi}{2} - \theta\right) + i \sin\left(\frac{\pi}{2} - \theta\right) \right)^n$$

$$= \cos\left(\frac{n\pi}{2} - n0\right) + i \sin\left(\frac{n\pi}{2} - n0\right) = \underline{\underline{R.H.S}}$$



OMG { MATHS }
The poetry of logical ideas.