

Trigonometry And Matrices : De Moivre's Theorem

$$x + \frac{1}{x} = 2 \cos \theta \quad \text{prove that } x^n + \frac{1}{x^n} = 2 \cos n\theta$$

Sol. $x + \frac{1}{x} = 2 \cos \theta$

$$x^2 + 1 = 2x \cos \theta$$

$$x^2 - 2x \cos \theta + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \cos \theta \pm \sqrt{4 \cos^2 \theta - 4(1)(1)}}{2(1)}$$

$$= \frac{2 \cos \theta \pm 2 \sqrt{\cos^2 \theta - 1}}{2(1)}$$

$$= \frac{2 (\cos \theta \pm \sqrt{(-1)(1 - \cos^2 \theta)})}{2}$$

$$= \cos \theta \pm \sqrt{i^2 \cdot \sin^2 \theta}$$

$$= \cos \theta \pm i \sin \theta.$$

$$x = (\cos \theta + i \sin \theta), (\cos \theta - i \sin \theta)$$

When

$$x = \cos \theta + i \sin \theta$$

$$x^n + \frac{1}{x^n} = (\cos \theta + i \sin \theta)^n + \frac{1}{(\cos \theta + i \sin \theta)^n}$$

$$= (\cos \theta + i \sin \theta)^n + (\cos \theta + i \sin \theta)^{-n}$$

$$= \cos n\theta + i \cancel{\sin n\theta} + \cos n\theta + i (-\cancel{\sin n\theta})$$

$$= 2 \cos n\theta = \underline{\underline{\text{R.H.S}}}$$

whence

$$x = \cos\theta - i \sin\theta$$

$$x^n + \frac{1}{x^n} = (\cos\theta - i \sin\theta)^n + \frac{1}{(\cos\theta - i \sin\theta)^n}$$

$$= \cos n\theta - i \sin n\theta + (\cos\theta - i \sin\theta)^{-n}$$

$$= \cos n\theta - i \sin n\theta + \cos n\theta + i \sin n\theta$$

$$= 2 \cos n\theta = \text{R.H.S.}$$

Hence
$$x^n + \frac{1}{x^n} = 2 \cos n\theta.$$



OMG { MATHS }
The poetry of logical ideas.