

Trigonometry And Matrices : De Moivre's Theorem

If α and β be the roots of $x^2 - 2x + 4 = 0$, Prove that

$$(i) \quad \alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}$$

$$(ii) \quad \alpha^6 + \beta^6 = 128$$

Sol.

$$x^2 - 2x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm 2\sqrt{1-4}}{2}$$

$$= \frac{2(1 \pm \sqrt{-3})}{2} = 1 \pm i\sqrt{3}$$

$$\alpha = 1 + i\sqrt{3} \quad \beta = 1 - i\sqrt{3}$$

Let $1 + i\sqrt{3} = r(\cos\theta + i\sin\theta)$

By Comparing.

$$r \cos \theta = 1 \quad - \textcircled{1}$$

$$r \sin \theta = \sqrt{3} \quad - \textcircled{2}$$

By squaring and adding

$$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 3$$

$$r^2 = 4$$

$$\boxed{r = 2.}$$

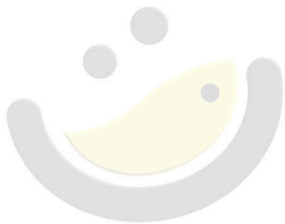
Divide $\textcircled{2}$ by $\textcircled{1}$

$$\frac{r \sin \theta}{r \cos \theta} = \frac{\sqrt{3}}{1}$$

$$\tan \theta = \sqrt{3}.$$

$$\underline{\underline{\theta = 60^\circ}}$$

$$\begin{aligned} \text{(i) } \alpha^n + \beta^n &= (r(\cos \theta + i \sin \theta))^n + (r(\cos \theta - i \sin \theta))^n \\ &= r^n (\cancel{\cos n\theta} + i \cancel{\sin n\theta} + \cancel{\cos n\theta} - i \cancel{\sin n\theta}) \\ &= r^n (2 \cos n\theta) \\ &= 2^n \cdot 2 \cos n(60^\circ) \\ &= 2^{n+1} \cos n\pi/3 = \text{R.H.S} \end{aligned}$$



$$\alpha^6 + \beta^6 = 2^{6+1} \cos \frac{6\pi}{3} \quad \text{Put } n=6$$

$$= 2^7 \cdot \cos 2\pi = 128 = \underline{\underline{R.H.S}}$$



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The poetry of logical ideas.