Exp. If α, β be roots of $t^2 - 2t + 2 = 0$ then prove that

also $x + 1 = \cot\phi$

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{\sin n\phi}{\sin^n \phi}$$

Sol.

$$t^2 - 2t + 2 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2}$$

$$= \frac{2 \pm 2\sqrt{1-2}}{2}$$

$$= \frac{\cancel{2}\left(1 \pm \sqrt{-1}\right)}{\cancel{2}} = 1 \pm i \qquad \{\because i^2 = -1\}$$

$$\alpha = 1+i \qquad \beta = 1-i$$

L.H.S

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} \text{ --- } \star$$

$$(x+\alpha)^n$$

$$x+\alpha = x+1+i$$

$$= \cot\phi + i$$

$$= \frac{\cos\phi}{\sin\phi} + i$$

$$= \frac{\cos\phi + i\sin\phi}{\sin\phi}$$

$$(x+\alpha)^n = \frac{(\cos\phi + i\sin\phi)^n}{\sin^n\phi} =$$

$$(x+\beta)^n = ?$$

$$x+\beta = x+1-i$$

$$= \cot\phi - i$$

$$= \frac{\cos\phi}{\sin\phi} - i$$

$$= \frac{\cos\phi - i\sin\phi}{\sin\phi}$$

$$(x+\beta)^n = \left[\frac{\cos\phi - i\sin\phi}{\sin\phi}\right]^n$$

$$= \frac{(\cos\phi - i\sin\phi)^n}{\sin\phi}$$

$$= \frac{\cos n\phi + i \sin n\phi}{\sin^n \phi} \quad \text{—} ① \qquad = \frac{\cos n\phi - i \sin n\phi}{\sin^n \phi} \quad \text{—} ②$$

$$\alpha - \beta = 1 + i - (1 - i) = 1 + i - 1 + i$$
$$= 2i \quad \text{—} (iii)$$

Put ①, (ii) and (iii) in A

$$\underline{\underline{L.H.S}} \quad \frac{\dfrac{\cos n\phi + i \sin n\phi}{\sin^n \phi} - \dfrac{(\cos n\phi - i \sin n\phi)}{\sin^n \phi}}{2i}$$

$$= \frac{\dfrac{\cancel{\cos n\theta} + i \sin n\theta - \cancel{\cos n\theta} + i \sin n\theta}{\sin^n \theta}}{2i}$$

$$= \frac{\dfrac{\cancel{2i} \sin n\theta}{\sin^n \theta}}{\cancel{2i}} = \frac{\sin n\theta}{\sin^n \theta} = \underline{\underline{R.H.S}}$$