Apply De-Moivre's theorem to find an equation whose roots are the n^{th} powers of the roots of the equation

$$x^2 - 2x\cos\theta + 1 = 0$$

Sol.

$$x^2 - 2x\cos\theta + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4\cdot 1\cdot 1}}{2}$$

$$x = \frac{2\cos\theta \pm \sqrt{4(\cos^2\theta - 1)}}{2}$$

$$= \frac{\cancel{2}\left\{\cos\theta \pm \sqrt{(-1)(1-\cos^2\theta)}\right]}{\cancel{2}}$$

$$= \cos\theta \pm \sqrt{i^2 \cdot \sin^2\theta}$$

$$x = \cos\theta \pm i\sin\theta.$$

Roots $= \underset{\alpha}{(\cos\theta + i\sin\theta)}, \underset{\beta}{(\cos\theta - i\sin\theta)}$

α^n, β^n Roots.

$$S = \alpha^n + \beta^n = (\cos\theta + i\sin\theta)^n + (\cos\theta - i\sin\theta)^n$$

$$= \cos n\theta + i\cancel{\sin n\theta} + \cos n\theta - i\cancel{\sin n\theta}$$

$$S = 2\cos n\theta$$

$$P = \alpha^n \cdot \beta^n = (\cos\theta + i\sin\theta)^n \cdot (\cos\theta - i\sin\theta)^n$$

$$= [(\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)]^n$$

$$= (\cos^2\theta - i^2\sin^2\theta)^n$$

$$= (\cos^2\theta + \sin^2\theta)^n = (1)^n = 1$$

Now equation is

$$x^2 - Sx + P = 0$$

$$x^2 - 2\cos n\theta\, x + 1 = 0 \quad \text{Ans.}$$