

Trigonometry And Matrices : De Moivre's Theorem

$$\cos\alpha + 2\cos\beta + 3\cos\gamma = 0 \quad \text{and} \quad \sin\alpha + 2\sin\beta + 3\sin\gamma = 0$$

Prove that

$$(i) \quad \cos 3\alpha + 8 \cos 3\beta + 27 \cos 3\gamma = 18 \cos(\alpha + \beta + \gamma)$$

$$(ii) \quad \sin 3\alpha + 8 \sin 3\beta + 27 \sin 3\gamma = 18 \sin(\alpha + \beta + \gamma)$$

Sol. $a = \cos\alpha + i \sin\alpha$

$$b = 2\cos\beta + i 2\sin\beta$$

$$c = 3\cos\gamma + i 3\sin\gamma$$

$$a+b+c = (\cos\alpha + 2\cos\beta + 3\cos\gamma) + i(\sin\alpha + 2\sin\beta + 3\sin\gamma)$$

$$= 0 + i0$$

$$a+b+c = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$(\cos\alpha + i\sin\alpha)^3 + (2\cos\beta + 2i\sin\beta)^3 + (3\cos\gamma + 3i\sin\gamma)^3$$

$$= 3(\cos\alpha + i\sin\alpha)^2(\cos\beta + i\sin\beta)$$

$$3(\cos\gamma + i\sin\gamma)$$

$$\begin{aligned} \cos 3\alpha + i \sin 3\alpha + 2^3 (\cos \beta + i \sin \beta)^3 + 27 (\cos \gamma + i \sin \gamma)^3 \\ = 18 \cos \alpha \cdot \cos \beta \cdot \cos \gamma \end{aligned}$$

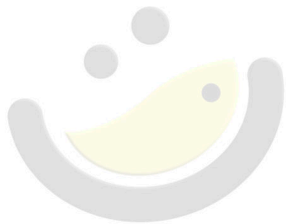
$$\begin{aligned} \cos 3\alpha + i \sin 3\alpha + 8 \cos 3\beta + 8i \sin 3\beta + 27 \cos 3\gamma + \\ 27i \sin 3\gamma = \\ 18 \cos (\alpha + \beta + \gamma) \end{aligned}$$

$$\begin{aligned} \cos 3\alpha + 8 \cos 3\beta + 27 \cos 3\gamma + i (\sin 3\alpha + 8 \sin 3\beta + \\ 27 \sin 3\gamma) = \\ 18 \{ \cos (\alpha + \beta + \gamma) + i \sin (\alpha + \beta + \gamma) \} \end{aligned}$$

$$\cos 3\alpha + 8 \cos 3\beta + 27 \cos 3\gamma = 18 \cos (\alpha + \beta + \gamma)$$

$$\sin 3\alpha + 8 \sin 3\beta + 27 \sin 3\gamma = 18 \sin (\alpha + \beta + \gamma)$$

Hence proved.



OMG! MATHS!
The poetry of logical ideas.