Trigonometry And Matrices : Introduction to De Moivre's Theorem

$$\frac{1}{\text{Cis}\,\theta} = \text{Cis}(-\theta) \qquad \text{Cis}\,\theta_1 \cdot \text{Cis}\,\theta_2 = \text{Cis}(\theta_1 + \theta_2)$$

$$\frac{\text{Cis}\,\theta_1}{\text{Cis}\,\theta_2} = \text{Cis}(\theta_1 - \theta_2)$$

$$\frac{1}{\text{Cis}\,\theta} = \text{Cis}(-\theta)$$

$\underline{\underline{\text{LHS}}}$

$$\frac{1}{\text{Cis}\,\theta} = \frac{1}{\cos\theta + i\sin\theta} \times \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$$

$$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta - (i\sin\theta)^2}$$

$$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta - i^2\sin^2\theta}$$

$$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta}. \qquad [\because i^2 = -1]$$

$$= \cos\theta - i\sin\theta = \text{cis}(-\theta)$$

$$= \underline{\underline{\text{R.H.S}}}$$

$$\frac{1}{\text{Cis}\,\theta} = \text{Cis}(-\theta)$$

$$\text{Cis}\,\theta_1 \cdot \text{Cis}\,\theta_2 = \text{Cis}(\theta_1 + \theta_2)$$

$\underline{\text{L.H.S}}$ $\quad (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$$\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + i^2\sin\theta_1\sin\theta_2$$

$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i[\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2] \qquad [\because i^2 = -1]$$

$$= \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$$

$$= \text{Cis}(\theta_1+\theta_2) = \underline{\underline{R.H.S}}$$

$$\frac{\text{Cis}\,\theta_1}{\text{Cis}\,\theta_2} = \text{Cis}(\theta_1-\theta_2)$$

$\underline{\underline{L.H.S}}$

$$\frac{\text{Cis}\,\theta_1}{\text{Cis}\,\theta_2} = \frac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2} \times \frac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$$

$$= \frac{\cos\theta_1\cos\theta_2 - i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 - i^2\sin\theta_1\sin\theta_2}{\cos^2\theta_2 - i^2\sin^2\theta_2}$$

$$= \frac{\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 + i(\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2)}{\cos^2\theta_2 + \sin^2\theta_2}$$

$[\because i^2 = -1]$

$$= \cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)$$

$$= \text{cis}(\theta_1 - \theta_2) = \text{R.H.S}$$