

## Trigonometry And Matrices : Introduction to De Moivre's Theorem

$$\frac{1}{\text{cis } \theta} = \text{cis}(-\theta)$$

$$\text{cis } \theta_1 \cdot \text{cis } \theta_2 = \text{cis}(\theta_1 + \theta_2)$$

$$\frac{\text{cis } \theta_1}{\text{cis } \theta_2} = \text{cis}(\theta_1 - \theta_2)$$

$$\frac{1}{\text{cis } \theta} = \text{cis}(-\theta)$$

L.H.S

$$\frac{1}{\text{cis } \theta} = \frac{1}{\cos \theta + i \sin \theta} \times \frac{\cos \theta - i \sin \theta}{\cos \theta - i \sin \theta}$$

$$= \frac{\cos \theta - i \sin \theta}{\cos^2 \theta - (i \sin \theta)^2}$$

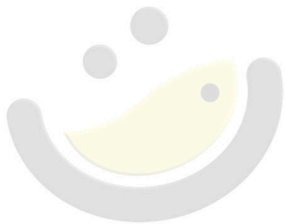
$$= \frac{\cos \theta - i \sin \theta}{\cos^2 \theta - i^2 \sin^2 \theta}$$

$$= \frac{\cos \theta - i \sin \theta}{\cos^2 \theta + \sin^2 \theta}$$

$$\{ \because i^2 = -1 \}$$

$$= \cos \theta - i \sin \theta = \cos(-\theta)$$

$$= \underline{\underline{\text{R.H.S}}}$$



$$\frac{1}{\operatorname{cis} 0} = \operatorname{cis}(-0)$$

$$\operatorname{cis} \theta_1 \cdot \operatorname{cis} \theta_2 = \operatorname{cis}(\theta_1 + \theta_2)$$

L.H.S  
=

$$(\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2)$$

$$\cos \theta_1 \cos \theta_2 + i \cos \theta_1 \sin \theta_2 + i \sin \theta_1 \cos \theta_2 + i^2 \sin \theta_1 \sin \theta_2$$

$$= \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 +$$

$$i [\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2]$$

$$[\because i^2 = -1]$$

$$= \cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)$$

$$= \text{cis}(\theta_1 + \theta_2) = \text{R.H.S}$$

$$\frac{\text{cis}\theta_1}{\text{cis}\theta_2} = \text{cis}(\theta_1 - \theta_2)$$

L.H.S

$$\frac{\text{cis}\theta_1}{\text{cis}\theta_2} = \frac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2} \times \frac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$$

$$= \frac{\cos\theta_1 \cos\theta_2 - i\cos\theta_1 \sin\theta_2 + i\sin\theta_1 \cos\theta_2 - i^2 \sin\theta_1 \sin\theta_2}{\cos^2\theta_2 - i^2 \sin^2\theta_2}$$

$$= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 + i(\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2)$$

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$$\cos^2\theta_2 + \sin^2\theta_2$$

$$= \cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)$$

[ $\because i^2 = -1$ ]

$$= \underline{\underline{\text{cis}(\theta_1 - \theta_2)}} = \underline{\underline{\text{R.H.S}}}$$

