

## Trigonometry And Matrices : Applications Of De Moivre's Theorem

Solve the equation  $x^5 - 1 = 0$  and show that the sum of  $n^{\text{th}}$  powers of the roots always vanishes unless  $n$  be a multiple of 5.  $n$  being an integer.

Sol.

$$x^5 - 1 = 0$$

$$x^5 = 1$$

$$x = (1)^{1/5}$$

$$x = (\cos 0 + i \sin 0)^{1/5}$$

$$= (\cos 2n\pi + i \sin 2n\pi)^{1/5}$$

$$= \cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5}$$

where  
 $n=0,1,2,3,4$

$$(\cos 0 + i \sin 0), \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right), \left(\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}\right)$$

$$\left(\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}\right), \left(\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}\right)$$

$n^{\text{th}}$  Power of Roots

$$(1)^n, \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right)^n, \left(\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}\right)^n,$$

$$\left(\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}\right)^n, \left(\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}\right)^n$$

$$1, \left(\cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5}\right), \left(\cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5}\right)^2$$

$$\left(\cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5}\right)^3, \left(\cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5}\right)^4$$

$$1, t, t^2, t^3, t^4$$

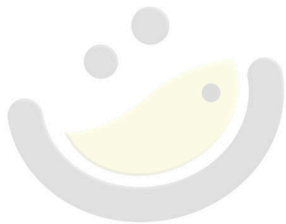
Case I  $n$  is not multiple of 5.

$$1 + t + t^2 + t^3 + t^4$$

$$\frac{1(1-t^5)}{1-t} = \frac{1 - \left(\cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5}\right)^5}{1-t}$$

$$= \frac{1 - (\cos 2n\pi + i \sin 2n\pi)}{1-t}$$

$$= \frac{1 - (1 + 0)}{1-t} = 0$$



Case II  $n$  is multiple of 5

$$\text{let } \underline{n = 5k}$$

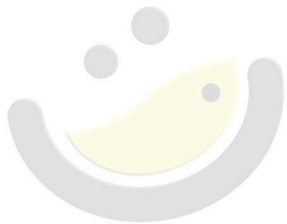
$$1 + t + t^2 + t^3 + t^4$$

$$1 + \left( \cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5} \right) + \left( \cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5} \right)^2$$

$$+ \left( \cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5} \right)^3 + \left( \cos \frac{2n\pi}{5} + i \sin \frac{2n\pi}{5} \right)^4$$

$$= 1 + \left( \cos 2k\pi + i \sin 2k\pi \right) + \left( \cos(2k\pi) + i \sin 2k\pi \right)^2$$

$$\begin{aligned} & + (\cos 2k\pi + i \sin 2k\pi)^3 + (\cos 2k\pi + i \sin 2k\pi)^4 \\ & = 1 + (1+0) + (1+0)^2 + (1+0)^3 + (1+0)^4 \\ & = 1 + 1 + 1 + 1 + 1 = \underline{\underline{5}} \text{ ans.} \end{aligned}$$



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