Limit and Continuity : Infinite Limits

Exp

using def. Prove that

$$\lim_{x \to \infty} \frac{e^x}{2e^x+3} = \frac{1}{2}$$

Sol.

$$f(x) = \frac{e^x}{2e^x+3} \qquad l = \frac{1}{2}$$

$$|f(x) - l| = \left| \frac{e^x}{2e^x+3} - \frac{1}{2} \right| = \left| \frac{\cancel{2e^x} - \cancel{2e^x} - 3}{2(2e^x+3)} \right|$$

$$|f(x)-\ell| = \frac{3}{2(2e^x+3)} \qquad \left[\because 2e^x+3>0\right]$$

$$|f(x)-\ell| < \epsilon$$

$$\text{if } \frac{3}{2(2e^x+3)} < \epsilon$$

$$\text{if } \frac{1}{2e^x+3} < \frac{2\epsilon}{3}$$

$$\text{if } 2e^x+3 > \frac{3}{2\epsilon}$$

if $\quad 2e^{x} > \dfrac{3}{2\epsilon} - 3$

if $\quad e^{x} > \dfrac{1}{2}\left(\dfrac{3}{2\epsilon} - 3\right)$

if $\quad x > \log\left[\dfrac{1}{2}\left(\dfrac{3}{2\epsilon} - 3\right)\right]$

$\therefore$ for given $\epsilon > 0 \quad \exists \quad M = \log\left[\dfrac{1}{2}\left(\dfrac{3}{2\epsilon} - 3\right)\right] > 0$

s.t. $|f(x) - l| < \epsilon$ for $x > M$.

$\therefore$ By def.

$$\lim_{x \to \infty} f(x) = l.$$

$$\lim_{x \to \infty} \frac{e^x}{2e^x+3} = \frac{1}{2}$$

Hence Proved